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Verification of Infinite-State Systems with Probabilistic Behaviour

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Abstract

We present a method for performing *quantitative analysis* of probabilistic systems with infinite state spaces: given an initial state s_{init}, a set F of final states, and a rational $\theta > 0$, compute a rational ρ such that the probability of reaching F from s_{init} is between and $\rho+\theta$. We present an algorithm working in breadth first manner to perform quantitative analysis of infinite state Markov chains, and provide sufficient conditions for termination of our algorithm.

Keywords: - Markov chains, quantitative analysis, lossy channel systems.

1. Introduction

The extension of the applicability of Model-checking to infinite state systems has become an interesting challenge. In [1] a general framework is presented for verification of infinite state transition systems based on the assumption that the transition relation is *monotonic* with respect to a *well-quasi ordering* on the set of states (configurations).

Recently, several attempts have been made for model checking of infinite state systems with *probabilistic behaviour*. The presence of probabilistic behaviour in some systems is due to the weights defined for the transitions that are sources for the dynamics of the system. Such weights cause the presence of probabilities of occurring configurations next to a specified one. The works in [2-7] consider *probabilistic lossy channel systems*: systems consisting of finite state processes communicating via channels which are unbounded and unreliable in the sense that they can non-deterministically lose messages. The motivation behind these works is that the probability of by which messages are lost inside the channels must be taken into account since we are dealing with unreliable communication, in spite of assigning weight to each of the transitions. In this paper, we extend the general framework of [1,8] from the context of transition systems to that of *Markov chains* (with potentially infinite state spaces). We consider the *quantitative analysis* problem: given an initial state s_{init}, a set F of final states, and a rational $\theta>0$ (corresponding to the precision of the analysis), compute a rational ρ such that the probability of reaching F from s_{init} is between ρ and $\rho+\theta$. To solve the above problem, we have designed an algorithm

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based on a simple procedure presented in [6] that constructs a reachability tree starting from s_{init} in a breadth-first manner. The main contribution of this paper is to provide sufficient conditions under which the algorithm is guaranteed to terminate.

These conditions are:

- The Markov chain is *finitely spanning*, i.e there is a natural number K such that, if a state s_2 is reachable from a state s_1, then s_2 is reachable from s_1 within K steps.
- The Markov chain is coarse, i.e, there is a rational $\alpha>0$ such that each enabled transition leads to a probability of change which is at least α.

Next, we relax the coarseness condition and consider those Markov chains which are *almost coarse*:

the probability of reaching a node violating the coarseness condition from the initial state s_{init} before reaching a closed node is less than θ. We show that our algorithm still terminates in case the Markov chain is almost coarse.

Related Work. The works in [3-5,7] all consider probabilistic lossy channel systems. However, these papers only consider *qualitative analysis*: is the probability of reaching F from s_{init} equal to one? [6] Considers quantitative analysis. However, no arguments are given for proving termination of the algorithm. Recently, Rabinovich has presented an algorithm in [2] for solving the quantitative analysis problem for probabilistic lossy channel systems. The algorithm solves both reachability and repeated reachability (in *repeated reachability* we compute the probability by which the set of final states is visited infinitely often). The algorithm of [2] is different from the one we present in this paper. Furthermore, it can not be extended to other classes of infinite state Markov chains such as probabilistic Petri nets. For instance, it relies on the existence of a finite *attractor* in the Markov chain. An *attractor* is a set of states which is reachable from each state in the Markov chain with probability which is equal to one. In the case of probabilistic lossy channel system, the set of states with empty channels form a finite attractor. However, in the case of probabilistic Petri net, finite attractors fail to exist, and therefore the algorithm of [2] is not applicable. On the other hand, our algorithm, based on a simple breadth-first procedure is much more

intuitive than the one presented in [2]. Our algorithm does not rely on the notions of attractors or even *strongly connected components* as is the case with all existing decidability proofs (known to us) of infinite state Markov chains. In fact, we regard showing that a small number of simple conditions are sufficient to guarantee the termination of the breadth-first algorithm as one of the main contributions of this paper.

2. Markov Chains

Definition 1. *A Markov chain M is an ordered pair (S,P) where S is a (potentially infinite) set of states and P is a mapping from $S\times S$ to the set of rational numbers in the interval $[0,1]$ such that*

$$\sum_{s'} P(s,s')=1 \; \forall s \in S.$$

A Markov chain induces a transition system $(S, \rightarrow)$ where the transition relation $\rightarrow$ is given by $s_1 \rightarrow s_2$ if and only if $P(s_1, s2)>0$. In this manner, concepts defined for transition systems can also be lifted to Markov chains. In the following we give a number of definitions which are concepts related to transition systems and thus to Markov chains.

Definition 2. *Let $(S,\rightarrow)$ be a transition system, and F be a set of final states of S, for $s\varepsilon$ S, we define distance dist(s) of s to be the minimum natural number k such that F is reachable from s within k steps, i.e there is a sequence $s_0, s_1, \ldots, s_k$ of states with $s=s_0$, $s_k \varepsilon F$ and $s_i \rightarrow s_{i+1}$ for i: $0\leq i<k$, denoted by $s\rightarrow^k F$. In case $s\varepsilon F^{\neg}$, we define dist(s) $=\infty$.*

Definition 3. *A transition system $(S, \rightarrow)$ is said to be effective if the following two conditions are satisfied*

- *Post is computable $\forall$ $s\in S$.*

- *Reachability is decidable in S.*

Definition 4. *A transition system $(S,\rightarrow)$ is said to be of span K if for each $s\in S$, we have dist(s)$\leq K$ or dist(s)$=\infty$. Moreover, we say that a transition system is finitely spanning if it is of span K.*

Definition 5. *We say that a transition system is monotonic (with respect to a preorder $\leq$) if F is upward closed, and, $s_1\rightarrow s_2$ and $s_1\leq s_3$ imply that there is s_4 with $s_3\rightarrow s_4$ and $s_2\leq s_4$.*

In [9], the following theorem is shown, which gives sufficient conditions for a transition system to be finitely spanning.

Theorem 1. *If a transition system is monotonic and well-quasi ordered then it is finitely spanning*

Definition 6. *Let $(S,\rightarrow)$ be a transition system, a computation π (from s_0) of S is an infinite sequence of states $s_0, s_1, \ldots$. We use $\pi(i)$ to denote s_i.*

Let F be a set of final states, we use ($s \models\Diamond F$) to denote the set

$\{\pi|\ \pi(0)=s$ and $\exists$ i. $\pi(i) \in F\}$

and we use ($s \models Q_1$ Before Q_2) for $Q_1, Q_2 \subseteq S$ to denote the set

$\{\pi \mid \pi(0)=s$ and $\exists$ j. $\pi(j) \in Q_1$ and $\forall$ $i<j$ $\pi(i) \notin Q_2\}$

A state s is said to be of coarseness α if $\forall$ $s' \in S$, $P(s,s')>0$ implies that $P(s,s') \geq \alpha$. A Markov chain $\boldsymbol{M}=(S, P)$ is said to be of coarseness α if each $s \in S$ is of coarseness α, and we say that $\boldsymbol{M}$ is *coarse* if $\boldsymbol{M}$ is of coarseness α for some $\alpha>0$. Notice that if $\boldsymbol{M}$ is coarse then the underlying transition system is finitely branching that is for any state s in the system the set *post(s)* is finite, however the converse is not necessarily true.

A Markov chain *(S,P)* induces a natural measure on the set of computations starting from any state $s \in S$. In the following we recall some basic notions from measure theory [10].

Definition 7: *A σ-algebra Δ over a set Ω is a subset of the set of all subsets of Ω (i.e Δ is a subset of the power set of Ω) such that the following three conditions hold:*

- $\Omega\in\Delta$
- *for any $A \in\Delta$, $A^c \in\Delta$ (A^c is the complement of A)*
- *for A, $B \in \Delta$, $A\cup B\in \Delta$*

Definition 8: *A measurable space is a pair (Ω,Δ) consisting of a non empty set Ω and a σ-algebra Δ of its subsets that are called measurable sets.*

Definition 9: *A probability measure defined on a measurable space (Ω,Δ) is a mapping Prob: $\Delta \rightarrow [0,1]$ such that Prob$(\Omega)=1$ and is countably additive, that is, for any $A, B \in \Delta$, Prob$(A\cup B)$=Prob(A)+Prob(B).*

Definition 10: *A probabilistic space is a triple $(\Omega,\Delta,Prob)$ where (Ω,Δ) is a measurable space and Prob is a probability measure defined on (Ω,Δ).*
Consider a state s in a Markov chain $M=(S,P)$. On the sets of computations that start at s, The probabilistic space $(\Omega,\Delta,Prob_M)$ is defined as follows:
Ω is the set of all infinite sequences of states starting from s, Δ is the σ-algebra generated by the basic cylindric sets D_u for every $u \in sS^*$, and the probability measure $Prob_M$ is defined by

$$Prob_M \quad (D_u)= \prod_{0,\dots,n-1} \quad P(s_i,s_{i+1}) \quad \text{where}$$

$u=s_0,s_1,\dots,s_n$.

3 Quantitative Analysis

We shall consider the following problem for Markov chains which is the main goal of this paper in hand:

Quantitative Analysis
Instance: A Markov chain $M=(S,P)$, a state $s_{init} \in S$, a set F of final states, and a rational $\theta > 0$.

Task: Compute a rational ρ such that $\rho \leq Prob_M(s_{init} \models \Diamond F) \leq \rho + \theta$
In other words, given a state s_{init} and a rational θ, compute a rational ρ such that the probability of reaching F from s_{init} lies between ρ and ρ+θ. The idea of the procedure presented in [6] is the same as the one we give here, but there has not been presented any condition for termination.We use $Yes^j(M,s_{init})$ to denote the value of the variable *Yes* after the algorithm has explored the reachability tree up to depth j (i.e any element (s,r) in *store* is such that $s_{init} \xrightarrow{j+1} s$), when the algorithm is run on Markov chain M with initial state s_{init}. We define $No^j(M,s_{init})$ in a similar manner.

Algorithm - Quantitative Analysis
Input
An effective Markov chain $M=(S,P)$, a state $s_{init} \in S$, a set of final states F, and a positive rational θ.
Output
a rational ρ such that the probability of reaching F from s_{init} is greater than or equal to ρ and less or equal to ρ+θ.
Variables
Yes, No:$\boldsymbol{Q}$ ($\boldsymbol{Q}$is the set of all rational numbers)
store: with elements in $S \times \boldsymbol{Q}$ initially empty
begin
1. add $(s_{init},1)$ to the end of *store*

2. **repeat**

3. remove (s,r) from *store*

4. if $s \in F$ then $Yes:=Yes+r$

5.else if $s \in F^{\sim}$ then $No:=No+r$

6. else

7. for each $s' \in post(s)$

8. add $(s',r*P(s,s'))$ to the end of store

9. **until** $Yes+No \geq 1-\theta$

10. **return**(*Yes*)
end

4 Correctness and Termination

In this section, we prove that the quantitative analysis algorithm terminates in case the Markov chain M is coarse and finitely spanning, so the quantitative analysis problem is solvable for effective, coarse and finitely spanning Markov chains.

Lemma 1. *The quantitative analysis algorithm terminates in case **M** is coarse and finitely spanning.*

Proof. Let K be the span of M, and α its coarseness. So any state s from which F is reachable (i.e $s \notin F^{-}$) satisfies $dist(s) \leq K$, so from any such state, F is reachable with probability at least α^K. Hence the set of paths avoiding $F \cup F^{-}$ forever has measure 0.so $Prob_M(s_{init} \models \Diamond F \cup F^{-})=1$. Therefore $\lim_{j\to\infty}(Yes^j+No^j)=1$. Hence we can reach a point where $Yes+No \geq 1-\theta$ for some positive rational θ which guarantees the termination of the algorithm.

Lemma 2. *If the quantitative analysis algorithm terminates at depth j then*
$Yes^j(\mathbf{M},s_{init}) \leq Prob_M(s_{init} \models \Diamond F) \leq Yes^j(\mathbf{M},s_{init})+\theta$

Proof. The variable Yes is the summation of the probabilities of those which will get into F eventually, so at depth j some paths may not have reached F, so it is straight forward that $Yes^j(\mathbf{M},s_{init}) \leq Prob_M(s_{init} \models \Diamond F)$ So is $No^j(\mathbf{M},s_{init})$, which is the summation of the probabilities of those computations that will never reach F, that is $No^j(\mathbf{M},s_{init}) \leq Prob_M(s_{init} \models F^{-}\,\underline{Before}\,F)$. From the inequality $Yes^j+No^j \leq 1$ we notice that
$Prob_M(s_{init} \models \Diamond F) \leq 1- Prob_M(s_{init} \models F^{-}\,\underline{Before}\,F)$. It follows that $Yes^j \leq Prob_M(s_{init} \models \Diamond F) \leq 1-No^j$
The result follows from the fact that $Yes^j+No^j \geq 1-\theta$ when the algorithm terminates.

From the above two lemmas we can state the following theorem

Theorem 2. *The quantitative analysis problem is decidable for effective, coarse and finitely spanning Markov chains.*

5. Almost Coarse Markov Chains

Let *M=(S,P)* be a Markov chain, and let $\alpha \in \mathbf{Q}$. We define *Bad(M,α)* to be the set:
$\{s \mid \exists\, s'.\ 0<P(s,s')<\alpha\}$

In other words, the states in *Bad(M,α)* are those which violate the coarseness condition.

Recall that $F^*=F \cup F^{-}$ Let $\alpha,\omega \in \mathbf{Q}$ with $\alpha>0$. We define *Coarse(M, α,ω)* to be the set:
{s| the probability of $s \models F^*$ <u>Before</u> Bad(M,α) is greater than or equal to 1-ω}

Definition 11. *Let **M** be a Markov chain with the initial state s_{init}, we say that **M** is almost coarse from s_{init} if and only if for any $\omega>0$, $\exists$ $\alpha>0$ such that $s_{init} \in Coarse(\mathbf{M},\alpha,\omega)$.*

We show that the quantitative analysis algorithm still terminates in case ***M*** is finitely spanning and $\exists$ $\alpha>0$ and $\omega<\theta$ such that $\underline{s_{init}} \in Coarse(\mathbf{M},\alpha,\omega)$.

The existence of α and ω guarantee the termination of the quantitative analysis algorithm by making little changes in the code, replacing line 5 in the code by the following two lines:

5a. else if $s \in F^{-}$ then $No:=No+r$

5b. else if $s \in Bad(\mathbf{M},\alpha)$ then $B:=B+r$

and replacing line 9 by:

9. **until** $Yes+No+B \geq 1+\omega-\theta$

Lemma 3. *The quantitative analysis algorithm terminates in case **M** is finitely spanning and $s_{init} \in Coarse(\mathbf{M},\alpha,\omega)$ for some α and ω <θ.*

Which concludes the following theorem

Theorem 3. *The quantitative analysis problem is decidable for effective, finitely spanning and almost coarse Markov chains.*

6. Probabilistic Petri Nets

We define a variant of Petri nets where each transition is assigned a weight defined by a natural number.

Definition 12. *The 5-tuple (P, T, I, O, w) where (P, T, I, O) is a Petri net and w:T→N is a function from the set of transitions to the set of natural numbers (i.e assigning to each transition a weight) is said to be probabilistic Petri net.*

The following demonstrates how to construct Markov chain from probabilistic Petri nets:

Let X be the set of all possible markings of the probabilistic Petri net $R=(P, T, I, O, w)$. for each $\mu\in X$ define:

enabled(μ) $=\{ t : t$ *is an enabled transition at* $\mu\}$.

We define $P:X\times X\to[0,1]$ by:

$$P(\mu 1,\mu 2\ ) = \frac{\sum_{\delta(\mu 1,t)=\mu 2} w(t)}{\sum_{t'\in enabled(\mu 1)} w(t')} \quad \text{if } \exists\ t\in T$$

$\delta(\mu_1,t)=\mu_2$, and 0 otherwise.

Note that δ is the next state function of the Petri net *(P, T, I, O)*. Now it is easy to show that *(X, P)* is a Markov chain which we prove as following:

The summation in the numerator is a part of that in the denominator, so $P(\mu_1, \mu_2)$ is always between 0 and 1 which is the first condition of Markov chain. For the second condition consider a marking μ with the set of its successors *post(μ)*, now adding all the probabilities $P(\mu,\mu')$ $\forall$ $\mu'\in$ *post(μ)* will result in a fraction with equal numerator and denominator. In the following we prove the sufficient conditions for the termination of the quantitative analysis algorithm for probabilistic Petri nets.

Theorem 4. *The Markov chain induced by a probabilistic Petri net is both coarse and finitely spanning*

Regarding the condition of being finitely spanning, it follows directly from the monotonicity of the wqo≤ on markings and the assumption that the set of final states is upward closed. Regarding the condition of coarseness let $\beta=min\{ w(t) \mid t \in T\}$, and let:

$$\alpha=\frac{\beta}{\sum_{t'} w(t')}$$

Now it is obvious that $P(\mu,\mu')\geq \alpha\ \forall\ \mu$ and μ', because the smallest positive value which the numerator takes is the minimum weight among all the weights assigned to the transitions, and the greatest value which the denominator attains is the summation of the weights of all the transitions. Depending on the above theorem we deduce that the quantitative analysis problem is solvable for probabilistic Petri nets.

7.Probabilistic Lossy Channel Systems

A *Probabilistic lossy channel system* ℓ is the tuple (S,C,M,T,λ,w) where (S,C,M,T) is an LCS, $\lambda\in[0,1]$, and w is a mapping from T to the set of positive natural numbers, in the following we derive the Markov chain induced by the above model:

First we compute probabilities of reaching states through loss of messages. For $x,y\in M^*$, we define $\#(x,y)$ to be the number of different ways by which we can delete symbols in word y in order to obtain x. also define:

$P_L(x,y)=\#(x,y)\ \lambda^{\#y-\#x}\ (1-\lambda)^{\#x}$

Also for w_1, w_2 (mappings from the set of channels to M*) define

$$P_L(w_1,w_2)=\prod_{c\in C} P_L(w_1(c),w_2(c))$$

We take $P_L((s_1,w_1),(s_2,w_2))= P_L(w_1,w_2)$ if $s_1=s_2$ and 0 otherwise, define

$$w(s,w)= \sum_{t\in enabled(s,w)} w(t)$$

$P_t((s_1,w_1), (s_2,w_2),x)$=if $t((s_1,w_1),x)$ is undefined then 0 else $(w(x)/w(s_1,w_1)).P_L(t((s_1,w_1),x),(s_2,w_2))$

and the probability function is defined as following:

$$P((s_1,w_1),(s_2,w_2))=\sum_{x\in T} P_t((s_1,w_1),(s_2,w_2),x)$$

Where the summation runs over all the t's in the system. The function t that has taken (s_1,w_1) as argument is defined in [7] After we have shown how the probabilistic lossy channel system can induce Markov chain. We can state that the quantitative analysis problem is solvable for probabilistic lossy channel systems due to almost coarseness of probabilistic lossy channel system which we will state in the following.

Theorem 5. *For an initial state s_{init} and $\omega>0$, $\exists\,\alpha>0$ such that $s_{init}\in Coarse(M,\alpha,\omega)$ where M is the Markov chain induced by a probabilistic lossy channel system.*

In the following, we describe the main ingredients for the proof. For any $j\in\aleph$, where $\aleph$ is the set of natural numbers, let $S_j=\{s \mid \#s=j\}$. The proof relies on the following two properties:

- Since each message inside a channel may be lost with a fixed probability λ during each step, the probability of losing messages increases with the size of the state. Starting from a state $s\in S_N$ for some $N\in\aleph$, the probability that we reach a state $s'\in N'$ with $N'\geq N$ decreases exponentially with N. This means that starting from any state, a computation tends to visit states with small sizes before reaching a state with a large size. In fact, starting from a state $s\in S_M$ for some $M\in\aleph$, the probability of reaching an initial state before reaching a state in S_N can be made arbitrarily close to one by increasing the value of N.

The above property implies that a computation with a high probability will return to the set of initial states "many times" before reaching a state with a large size .

- Let K be the span of M. We know that for each initial state s_{init}, either $s_{init}\in F^{\sim}$ or s_{init} is at distance (at most) K from F^*. Therefore, there is positive $\mu\in Q$ such that any computation starting from s_{init} will visit F^* before visiting S_K with a probability which is at least μ. This means that each time a computation returns to an initial state, the probability of reaching F^* before reaching a large state (with size greater than K) is bounded below.

Together, the two properties imply that starting from an initial state, we can make the probability of reaching F^* before S_N arbitrarily large by increasing the value of N. This gives the result, since we can now define α to be the smallest positive rational such that there are states s, s' with $s\in S$ where

$$S=\bigcup_{0\leq i\leq N} Si \text{ and } P(s,s')=\alpha$$

The following lemma shows the first property, namely for a state $s\in S_M$, we can make the probability of reaching an initial before S_N arbitrarily close to one by increasing the value of N.

Lemma 4. *Consider $M\in\aleph$, $s\in S_M$, and $\omega\in Q$ with $\omega>0$. There is an $N\in\aleph$ such that $Prob_M(s\models S_0\ \underline{Before}\ S_N)\geq 1-\omega$*

*Proof. We consider an abstraction of **M** (figure 1.) which is a Markov chain $M_A=(S_A,P_A)$, where $S_A=\{0,1,2,\ldots\}$ and P_A is defined as follows:*

- $P_A(i,i+1)=v_i$ for $i>0$, where v_i is the largest rational such that there is a state $s\in S_i$ with

$$\sum_{s'\in Si\cup Si+1} P(s,s')=v_i$$

In other words, v_i is the largest probability by which, if we start from a *state in S_i, then we do not decrease the size in the next step.*

- $P_A(i,i-1)=\mu_i = 1- \nu_i$ *for* $i > 0$.
- $P_A(i,j)=0$ *otherwise.*

Intuitively, a state i in M_A *represents an abstraction of the set* S_i *in* **M**. *The claim of the lemma follows from the following two properties of* M_A:

- *For natural numbers* $M_1<M_2<M_3$ *and* $s\in S_{M2}$, *we have*

$Prob_M(s \models S_{M1}$ Before $S_{M3}) \geq Prob_{MA}(M_2 \models M_1$ Before $M_3)$

In other words, the Markov chain M_A *tends to move to the left less than* **M**. *This follows immediately from the definition of* M_A. *There is an N such that*

$Prob_{MA}(M \models 0$ Before $N) \geq 1- \omega$

To show this, consider $s\in S_i$. *We know that the probability of not decreasing the size of the channel is at most*

$(1-\lambda)^{i+1}+(i+1)\,\lambda\,(1-\lambda)^i$

We know that for larger values of i, the ratio μ_i/ν_i *gets arbitrarily large. We let L be the least index for which* $i\geq L$ *implies* $\mu_i/\nu_i \geq 2$. *In [11] it is shown that*

$$Prob_{MA}(M \models 0 \text{ Before } N)=\frac{\sum_{i=M}^{N-1}\rho i}{\sum_{i=0}^{N-1}\rho i}$$

where

$$\rho i = \frac{\prod_{j=1}^{i}\mu j}{\prod_{j=1}^{i}\nu j}$$

We notice that for $i \geq L$, *we have* $\rho i \geq \gamma\, 2^{i-}$

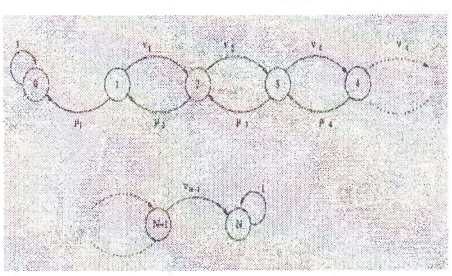

Fig. 1. The Markov chain Ma

$$\text{where } \rho i = \frac{\prod_{j=1}^{L}\mu j}{\prod_{j=1}^{L}\nu j}$$

We conclude that by choosing N to be sufficiently large, the value of $\sum_{i=L}^{N-1}\rho i$ *can be made arbitrarily large, i.e we can get* $Prob_{MA}(M \models 0$ Before $N)$ *arbitrarily close to one.*

***Proof of Theorem** 5.* Let μ be a positive rational such that, for each initial state *s*, there is a sequence $s_0,s_1,\ldots,s_l$, where $0\leq l\leq K, \mu \leq \prod_{i=0}^{l-1} P(s_i, s_{i+1})$, $s=s_0$, and $s_l \in F^*$

The rational μ exists since *K* is the span of **M**. Notice that $s_i \notin S_K$ for i: $0\leq i\leq l$, and that μ is a lower bound on the measure of computations that originate from an initial state and reach F^* before reaching S_K.

We consider four sets of states, namely F^*, S_0, S_K and S_N where $N\in \aleph$ and $N>K$. Consider an initial state $s\in S_0$, the state *s* satisfies the foloowing properties:

- $Prob_M\ (s \models \Diamond\ (F^*\cup\ S_K))=1$: a computation that starts from *s* and does not visit S_K, it will visit F^* by probability one by similar reasoning to lemma 4.
- $Prob_M(s \models F^*$ Before $S_K) \geq \mu$: follows from definition of μ.
- $Prob_M(s \models S_N$ Before $S_K)$ =0: since $N>K$.

Let ν be a rational such that $Prob_M(s \models S_0$ Before $S_K) \geq \nu$ for each $s\in S_K$. Such a ν exists, since S_K is finite and since for each state $s \in S_K$, $(s \models S_0$ Before $S_K)$ holds with non-zero probability. This means that each $s\in S_K$ satisfies the following properties:

- $Prob_M(s \models \Diamond(S_0 \cup S_N))=1$
- $Prob_M(s \models\ S_0$ Before $S_N) \geq \nu$: by definition of ν.

From the properties stated above, we define a new abstract Markov chain shown in Figure 2. $M_B=(S_B,P_B)$, where $S_B=\{0,1,2,3\}$. Intuitively, the states 0,1,2, and 3 represent the abstractions of the sets F^*, S_0, S_K, and S_N respectively. The probability function P_B is defined by the labels on the arcs shown in the figure. From the properties of S_0 and S_K described above, it follows immediately that for each $s \in S_0$

$Prob_{MB}(1 \models 0 \underline{Before}\ 3) \leq Prob_M(s \models F^* \underline{Before}\ S_N)$

From the definition of M_B it is straightforward to verify that

$$Prob_{MB}(1 \models 0 \underline{Before}\ 3) = \frac{\mu}{(1-\nu)(1-\mu)}$$

By lemma 4. and the definition of v, we can choose N to make v arbitrarily close to one. In particular we can choose N sufficiently large such that:

$$\frac{\mu}{(1-\nu)(1-\mu)} \geq 1-\omega$$

We can now define α to be the smallest positive rational such that there are states s, s' with

$s \in \bigcup_{0 \leq i \leq N} Si$ and $P(s, s')=\alpha$

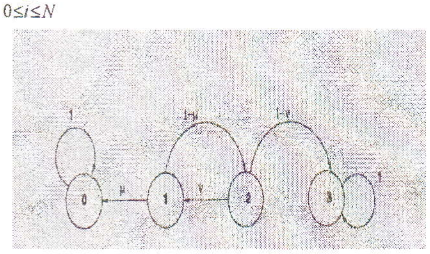

Fig. 2. Simplified Markov chain Mb

8 .Conclusion.

We have presented a methodology for solving the quantitative analysis problem for a class of infinite state Markov chains. The method is based on a simple breadth-first procedure presented in [6] giving sufficient conditions to guarantee the termination of the procedure. In this paper, we have considered two classes namely probabilistic Petri nets and probabilistic lossy channel systems. We have concluded that the quantitative analysis is decidable for both models by proving that the Markov chains induced by these two models satisfy the conditions for termination.

There are two interesting directions for future research.

- The extension of the method concluded in this work in order to carry out the quantitative analysis of liveness properties rather than safety properties.
- In this work, we have concluded the decidability of quantitative analysis of probabilistic lossy channel systems, while in [4] the decidability of qualitative analysis is concluded for it. On the other hand, we have defined a probabilistic version of Petri nets and concluded the decidability of quantitative analysis for it. The remaining challenge is to conclude the decidability or undecidability of the qualitative analysis for probabilistic Petri nets.

References

1- Abdulla P.A., Cerans K., Jonsson B., and Yih-Kuen T. Algorithmic analysis of programs with well-quasi ordered domains. *Information and Computation,* **2000,** 160:109-127.

2-Rabinovich A. Quantitative analysis of probabilistic lossy channel systems. In *Proc. ICALP'03,* **2003**.

3- Bertrand N and Snoebelen Ph. Model checking lossy channel systems is probably decidable. In *Proc. FOSSACSO3,* **2003,** 2620.

4-Abdulla P. A. and Rabinovich A. Verification of Probabilistic systems with faulty communication. In *Proc. FOSSACSO3,* **2003** 2620.

5- Baier C. and Engelen B. Establishing qualitative properties for probabilistic lossy channel systems. In Katoen, editor, *ARTS'99,* **1999,** 1601 of *Lecture Notes in Computer Science,* pages 34-52.

6- Iyer P. and Narasimha M. Probabilistic lossy channel systems. In *TAPSOFT'97,* **1997.** volume 1214 of *Lecture Notes in Computer Science,* pages 667-681.

7- Abdulla P. A., Iyer P, and Jonsson B. Reasoning about probabilistic lossy channel systems. In C. Palamidessi, *Proc. CONCUR 2000, 11th Int. Conf. on Concurrency theory,* **1997,** 877 of *Lecture Notes in Computer Science.*

8- Finkel A. and Snoebelen Ph. Well-structured transition systems everywhere. April **1998,** Technical LSV-98-4. Ecole Normale Superieure de Cachan.

9-Abdulla P. A., Cerans K, Jonsson B, and Yih-Kuen T. General decidability theorems for infinite-state systems. **1996,** In *Proc. LICS'96 11th IEEE Int. Symp. On Logic in Computer Science,* pages 313-321.

10- Kemeny J. G., Snell J. L., and Knapp A. W.. *Denumerable Markov chains.* D van Nostad Co. **1966**

11- Karlin S. *A first Course in Stochastic Processes.* Academic Press. **1966.**

شیگاری بڕی سیستمە ئەگەرییەکان بە بۆشاییەکی ناکۆتا

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پوختە

رێگەیەک پێشکەش دەکەین بۆ ئەنجامدانی شیکاری بڕی سیستمە ئەگەرییەکان بە بۆشاییەکی نا کۆتا: ئەگەر حاڵەتێکی سەرەتای درابێت (S_{init}) وە هەروەها کۆمەڵەی (F) حاڵەتەکانی دوای و ژمارەیەکی ڕێژەیی (θ)، ژمارەیەک دەردەهێنین (ρ) بە جۆرێک ئەگەری گەیشتن بە (F) لە (S_{init}) لە نێوانی (ρ) و ($\rho+\theta$) بێت. خوارزمیەک پێشکەش دەکەیب کە بە جۆرێکی بەرین کار دەکات بۆ ئەنجامدانی شیکاری بڕی زنجیرەی مارکۆفی ناکۆتا، وە مەرجی پێویست دەدەین بۆ کۆتای هاتنی خوارزمییەکەمان

تحليل المنظومات اللامتناهية الحالات ذات الطبيعة الاحتمالية

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الخلاصة

سوف نقدم طريقة لقيام بتحليل الكمي لانظمة الاحتمالية ذات فضاء لا متنناهي ؛ معطيا حالة الاولية (S_{init}) مجموعة (F) عن حالات النهائية و عدد النسبي (θ) نحسب العدد النسبي (ρ) بحيث تكون احتمالية الوصول (F) من (S_{init}) بين (ρ) و ($\rho+\theta$). سنقدم خوارزمية تعمل بشكل الاتساعي للتحليل الكمي لسلاسل الماركوف اللامتناهية ، ونقدم شروطاً لازمة لانتهاء الخوارزمية .

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