



## Contra $(i, j) - S_C -$ Continuous Function

Hardi N. Aziz

Sulaimani University, Faculty of Science and Science Education, School of Science Education, Sulaimani, Kurdistan Region – Iraq

Harde.1984@yahoo.com

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### Abstract

In this paper, a new type of continuity in Bitopological spaces has been introduced, namely Contra  $(i, j) - S_C -$  continuity. Also, the relationship between this type of continuity and several other known type of continuity has been dealt with.

### 1. Introduction

In [4] Kelley was first introduced the concept of bitopological spaces, where two topologies  $\tau_1$  and  $\tau_2$  are defined on a nonempty set  $X$ . Throughout this paper,  $(X, \tau_1, \tau_2)$  is a bitopological space, and for any subset  $A$  of  $X$ ,  $i - \text{Int}(A)$  and  $i - \text{Cl}(A)$  denote, respectively, the interior and closure of  $A$  with respect to the topology  $\tau_i$  on  $X$ . However, to avoid cumbersome expressions, let  $i$  and  $j$  refer to the first and second topologies on  $X$ , respectively.

A subset  $A$  of a space  $(X, \tau)$  is called semi-open [3], (resp., regular open [1]) if  $A \subseteq \text{Cl}(\text{Int}(A))$  (resp.,  $A = \text{Int}(\text{Cl}(A))$ ). The complement of a semi-open (resp., regular open) set is said to semi-closed (resp., regular closed). The intersection of all semi – closed sets of  $X$  containing  $A$  is called semi – closure of  $A$  [8].

Carnahan D.A. in [9], introduced the concept of  $R -$  map and Dontchev J. and Noiri T. in [7], gave definition of contra semi – continuous functions. Alias B. K. et al in [10], defined

$(i, j) - S_C -$  continuity. These definitions will be mentioned in the preliminaries section. Here, in this article a new type of continuity pursued, which is contra  $(i, j) - S_C -$  continuity.

### 2. Preliminaries

We give the following definition and results which are used in next sections.

**Definition 2.1 [2]:** A topological space  $X$  is said to be  $T_1 -$  space, if to each pair of distinct points  $x, y$  of  $X$ , there exist a pair of open sets, one containing  $x$  but not  $y$  and the other containing  $y$  but not  $x$ .

**Definition 2.2 [5]:** A subset  $A$  of a bitopological space  $(X, \tau_1, \tau_2)$  is said to be  $(i, j) - S_C$  - open, if  $A$  is  $j$  - semi - open and for all  $x \in A$ , there exist an  $i$  - closed set  $F$  such that  $x \in F \subseteq A$ . A subset  $B$  of  $X$  is called  $(i, j) - S_C$  - closed if and only if  $B^c$  is  $(i, j) - S_C$  - open. The family of  $(i, j) - S_C$  - open (resp.,  $(i, j) - S_C$  - closed) subset of  $X$  is denoted by  $(i, j) - S_C O(X)$  (resp.,  $(i, j) - S_C C(X)$ ).

**Definition 2.3 [6]:** The set  $\cap \{U \in \tau: S \subseteq U\}$  is called the kernel of  $S$  and it is denoted by  $Ker(S)$ .

**Definition 2.4 [5]:** If  $A$  is a subset of a bitopological space  $(X, \tau_1, \tau_2)$ , then the  $(i, j) - S_C$  -interior  $((i, j) - S_C Int(A))$ ,  $(i, j) - S_C$  - closure  $((i, j) - S_C Cl(A))$ , and  $(i, j) - S_C Bd(A)$  are defined as:

- 1-  $(i, j) - S_C Int(A) = \cup \{U: U \subseteq A, U \in (i, j) - S_C O(X)\}$ .
- 2-  $(i, j) - S_C Cl(A) = \cap \{F: A \subseteq F, X - F \in (i, j) - S_C O(X)\}$ .
- 3-  $(i, j) - S_C Bd(A) = [(i, j) - S_C Cl(A)] - [(i, j) - S_C Int(A)]$ .

**Definition 2.5 [2]:** A space  $X$  is regular if for each  $x \in X$  and each open set  $G$  containing  $x$ , there exists an open set  $H$  such that  $x \in H \subseteq Cl(H) \subseteq G$ .

**Definition 2.6 [9]:** A function  $f: X \rightarrow Y$  is called  $R$  - map, if the inverse image of each regular open subset of  $Y$  is regular open in  $X$ .

**Definition 2.7 [2]:** A space  $X$  is said to be Urysohn, if for each two distinct points  $x, y$  in  $X$ , there exist two open sets  $U, V$  such that  $x \in U, y \in V, y \notin U$  and  $x \notin V$  and  $Cl(U) \cap Cl(V) = \emptyset$ .

**Definition 2.8 [7]:** A function  $f: X \rightarrow Y$  is called contra-semi continuous, if the preimage of every open subset of  $Y$  is semi-closed in  $X$ .

**Lemma 2.9 [6]:** The following properties hold for subsets  $U, V$  of a space  $X$ :

- 1)  $x \in ker(U)$  if and only if  $U \cap F \neq \emptyset$ , for any closed subset  $F$  containing  $x$ .
- 2)  $U \subseteq ker(U)$  and  $U = ker(U)$ , if  $U$  is open set.
- 3) If  $U \subseteq V$ , then  $ker(U) \subseteq ker(V)$ .

**Lemma 2.10 [5]:** Let  $(X, \tau_1, \tau_2)$  be a bitopological space. If  $(X, \tau_i)$  is  $T_1$  - space, then  $(i, j) - S_C O(X) = j - SO(X)$ .

**Theorem 2.11 [5]:** Let  $X_1, X_2$  be two bitopological spaces. If  $A \in (i, j) S_C O(X_1)$  and  $B \in (i, j) - S_C O(X_2)$ . Then  $A \times B \in (i, j) - S_C O(X_1 \times X_2)$ .

**Theorem 2.12 [5]:** Let  $Y$  be a subspace of a bitopological space  $(X, \tau_1, \tau_2)$  and  $A \subseteq Y$ , if  $A \in (i, j) S_C O(Y)$  and  $Y \in i - RO(X) \cap j - RO(X)$ , then  $A \in (i, j) - S_C O(X)$ .

**Definition 2.13 [10]:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is called  $(i, j) - S_C$  - continuous at a point  $x \in X$ , if for each  $j$  - open  $V$  of  $Y$  containing  $f(x)$ , there exists an  $(i, j) - S_C$  - open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq V$ .

If  $f$  is  $(i, j) - S_C$  - continuous at every point  $x$  of  $X$ , then it is called  $(i, j) - S_C$  - continuous.

### 3. Contra $(i, j) - S_C$ - continuous

**Definition 3.1:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be Contra  $(i, j) - S_C$  - continuous, if the inverse image of every  $j$  - open set of  $Y$  is  $(i, j) - S_C$  - closed set in  $X$ .

Contra  $(i, j) - S_C$  - continuous is clearly  $j$  - contra semi - Continuous. However, the converse does not hold in general, to see this consider the following example:

**Example 3.2:** Let  $X = \{a, b, c, d\}, \tau_1 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}, \tau_2 = \{\emptyset, \{a\}, \{a, b\}, X\}$  and  $Y = \{x, y, z, w\}, \sigma_1 = \{\emptyset, \{y\}, Y\}, \sigma_2 = \{\emptyset, \{x\}, Y\}$ .

Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function defined by;  $f(a) = w, f(b) = x = f(d),$

$f(c) = z.$  Then  $f$  is  $j$  - Contra semi - continuous function, but is not Contra  $(i, j) - S_C$  - continuous, Since  $f^{-1}(\{x\}) = \{b, d\} \notin (i, j) - S_C C(X).$

**Corollary 3.3:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is said to be Contra  $(i, j) - S_C$  - continuous, if the inverse image of every  $j$  - closed set of  $Y$  is  $(i, j) - S_C$  - open set in  $X$ .

**Theorem 3.4:** For a function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2),$  the following statements are equivalent:

- 1)  $f$  is Contra  $(i, j) - S_C$  - continuous.
- 2) The inverse image of every  $j$  - closed set of  $Y$  is  $(i, j) - S_C$  - open set in  $X$ .
- 3) For each  $x \in X$  and each  $j$  - closed set  $B$  in  $Y$  with  $f(x) \in B,$  there exists an  $(i, j) - S_C$  -open set  $A$  in  $X$  such that  $x \in A$  and  $f(A) \subseteq B.$
- 4)  $f((i, j) - S_C Cl(A)) \subseteq j - ker(f(A)),$  for every subset  $A$  of  $X.$
- 5)  $(i, j) - S_C Cl(f^{-1}(A)) \subseteq f^{-1}(j - ker(A)),$  for every subset  $A$  of  $Y.$

**Proof:**

**1  $\Rightarrow$  3:** Let  $x \in X$  and  $B$  be  $j$  - closed set in  $Y$  with  $f(x) \in B.$  By(1), it follows that

$f^{-1}(Y - B) = X - f^{-1}(B)$  is  $(i, j) - S_C$  - closed set in  $X,$  so  $f^{-1}(B)$  is  $(i, j) - S_C$  - open set in  $X.$  Take  $A = f^{-1}(B),$  we obtain  $x \in A$  and  $f(A) \subseteq B.$

**3  $\Rightarrow$  2:** Let  $B$  be  $j$  - closed set in  $Y$  with  $x \in f^{-1}(B).$  Since  $f(x) \in B,$  by(3), there exists an  $(i, j) - S_C$  - open set  $A$  in  $X$  containing  $x$  such that  $f(A) \subseteq B.$  It follows that

$x \in A \subseteq f^{-1}(B).$  Therefore,  $f^{-1}(B)$  is  $(i, j) - S_C$  - open set in  $X.$

**2  $\Rightarrow$  1:** Follows from **Corollary 3.3.**

**2  $\Rightarrow$  4:** Let  $A$  be any subset of  $X$  and let  $y \notin j - ker(f(A)).$  Then by **Lemma 2.9** there exists  $j$  - closed set  $F$  containing  $y$  such that  $f(A) \cap F = \emptyset.$  Hence,  $A \cap f^{-1}(F) = \emptyset,$  since  $f^{-1}(F)$  is  $(i, j) - S_C$  - open, then  $(i, j) - S_C Cl(A) \cap f^{-1}(F) = \emptyset.$  So,  $f((i, j) - S_C Cl(A)) \cap F = \emptyset$  and hence  $y \notin f((i, j) - S_C Cl(A)).$  Thus

$$f((i, j) - S_C Cl(A)) \subseteq j - ker(f(A)).$$

**4  $\Rightarrow$  5:** Let  $B$  be any subset of  $Y.$  Then by(4),  $f((i, j) - S_C Cl(f^{-1}(B))) \subseteq j - ker(B)$  and hence,  $(i, j) - S_C Cl(f^{-1}(B)) \subseteq f^{-1}(j - ker(B)).$

**5  $\Rightarrow$  1:** Let  $B$  be any  $j$  - open set in  $Y.$  Then by(5),  $(i, j) - S_C Cl(f^{-1}(B)) \subseteq f^{-1}(j - ker(B)).$

By **Lemma 2.9**  $f^{-1}(j - \ker(B)) = f^{-1}(B)$ , so

$(i, j) - S_C Cl(f^{-1}(B)) \subseteq f^{-1}(B)$  and  $f^{-1}(B) \subseteq (i, j) - S_C Cl(f^{-1}(B))$  always.

Therefore,  $(i, j) - S_C Cl(f^{-1}(B)) = f^{-1}(B)$ . We obtain  $f^{-1}(B)$  is an  $(i, j) - S_C -$  closed in  $X$ .

Thus,  $f$  is contra  $(i, j) - S_C -$  continuous.

**Proposition 3.5:** If a function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous and  $(Y, \sigma_j)$  is regular, then  $f$  is  $(i, j) - S_C -$  continuous.

**Proof:**

Let  $x$  be any point in  $X$  and  $V$  be any  $j -$  open set of  $Y$  containing  $f(x)$ . Since  $(Y, \sigma_j)$  is regular, so there exists  $j -$  open set  $G$  in  $Y$  containing  $f(x)$  such that  $j - cl(G) \subseteq V$ . Since  $f$  is contra  $(i, j) - S_C -$  continuous, so by **Theorem 3.4** there exists an  $(i, j) - S_C -$  open set  $U$  of  $X$  containing  $x$  such that  $f(U) \subseteq j - Cl(G) \subseteq V$ . Hence  $f(U) \subseteq V$ . Therefore  $f$  is  $(i, j) - S_C -$  continuous.

In the following result, shows that  $(i, j) - S_C -$  continuous need not be Contra  $(i, j) - S_C -$  continuous:

**Example 3.6:** Let  $X = \{a, b, c, d\}, \tau_1 = \{\emptyset, \{b\}, \{c\}, \{b, c\}, X\}, \tau_2 = \{\emptyset, \{a\}, \{a, b\}, X\}$  and  $Y = \{x, y, z, w\}, \sigma_1 = \{\emptyset, \{y\}, Y\}, \sigma_2 = \{\emptyset, \{x\}, Y\}$ .

Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function defined by  $f(a) = f(b) = f(d) = x$ , and  $f(c) = z$

Then  $f$  is  $(i, j) - S_C -$  continuous but not contra  $(i, j) - S_C -$  continuous, since

$$f^{-1}(\{x\}) = \{a, b, d\} \notin (i, j) - S_C C(X).$$

**Proposition 3.7:** If  $(X, \tau_i)$  is  $T_1 -$  space, then the function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous if and only if it is  $j -$  contra semi - continuous.

**Proof:** Follows from **Lemma 2.10**.

**Proposition 3.8:** If a function  $f: (X, \tau_1, \tau_2) \rightarrow (\prod Y_k, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous, then  $g_k of: (X, \tau_1, \tau_2) \rightarrow (Y_k, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous, for each  $k \in I$ , where  $g_k$  is the  $j -$  projection of  $\prod Y_k$  onto  $Y_k$ .

**Proof:** Let  $V_k$  be any  $j -$  open set in  $Y_k$ . Since  $g_k$  is  $j -$  continuous, so  $g_k^{-1}(V_k)$  is  $j -$  open in  $\prod Y_k$ . Since  $f$  is contra  $(i, j) - S_C -$  continuous, so  $f^{-1}(g_k^{-1}(V_k)) = (g_k of)^{-1}(V_k)$  is

$(i, j) - S_C -$  closed set in  $X$ . Therefore  $g_k of$  is contra  $(i, j) - S_C -$  continuous, for each  $k \in I$ .

**Proposition 3.9:** If a function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous and  $g: (Y, \sigma_1, \sigma_2) \rightarrow (Z, \eta_1, \eta_2)$  is  $j - R -$  map, then  $gof$  is contra  $(i, j) - S_C -$  continuous.

**Proof:** Let  $V$  be any  $j -$  regular open set in  $Z$ . Since  $g$  is  $j - R -$  map, so  $g^{-1}(V)$  is  $j -$  regular open set in  $Y$  and since every regular open set is open set, so  $g^{-1}(V)$  is  $j -$  open set in  $Y$ . Since  $f$  is contra  $(i, j) - S_C -$  continuous, so  $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $(i, j) - S_C -$  closed set in  $X$ . Therefore,  $gof$  is contra  $(i, j) - S_C -$  continuous.

**Proposition 3.10:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be a function. Then  $f$  is not contra  $(i, j) - S_C -$  continuous at  $x \in X$  if and only if  $x$  is an  $(i, j) - S_C -$  boundary point of the inverse image of a  $j -$  closed set in  $Y$  which containing  $f(x)$ .

**Proof: If Part:** Suppose that  $f$  is not contra  $(i, j) - S_C -$  continuous at  $x \in X$ . Then there exists  $j -$  closed set  $F$  in  $Y$  such that  $f(U) \cap (Y - F) \neq \emptyset$ , for every  $(i, j) - S_C -$  open set  $U$  in  $X$ , implies that  $U \cap f^{-1}(Y - F) \neq \emptyset$ , therefore we have,

$x \in (i, j) - S_C Cl(f^{-1}(Y - F)) = (i, j) - S_C Cl(X - f^{-1}(F))$ . However, since  $x \in f^{-1}(F)$ , so  $x \in (i, j) - S_C Cl(f^{-1}(F))$ . Therefore,  $x \in (i, j) - S_C Bd(f^{-1}(F))$ .

**Only if part:** Suppose that  $x \in (i, j) - S_C Bd(f^{-1}(F))$ , for some  $j -$  closed set  $F$  of  $Y$ . Now, let  $f$  is contra  $(i, j) - S_C -$  continuous at  $x$ . Then by **Theorem 3.4**, there exists an

$(i, j) - S_C -$  open set  $U$  of  $X$  such that  $f(U) \subseteq F$ . Therefore,  $x \in U \subseteq f^{-1}(F)$  and hence

$x \in (i, j) - S_C Int(f^{-1}(F))$ , so  $x \notin (i, j) - S_C Cl(X - f^{-1}(F))$ , and hence ,

$x \notin (i, j) - S_C Bd(f^{-1}(F))$ , which is contradiction. Therefore  $f$  is not contra  $(i, j) - S_C -$  continuous.

**Proposition 3.11:** Let  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be contra  $(i, j) - S_C -$  continuous. If  $Y$  is

$j -$  closed subset of a bitopological space  $(Z, \eta_1, \eta_2)$ , then  $f: (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$  is contra  $(i, j) - S_C -$  continuous.

**Proof:** Let  $V$  be any  $j -$  closed set in  $Z$ . Then  $V \cap Y$  is  $j -$  closed set in  $Y$ . Since

$f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous, so by **Corollary 3.3**  $f^{-1}(V \cap Y)$  is  $(i, j) - S_C -$  open set in  $X$ , but  $f(x) \in Y$ , for each  $x \in X$  and thus  $f^{-1}(V) = f^{-1}(V \cap Y)$  is an  $(i, j) - S_C -$  open set in  $X$ . Therefore by **Corollary 3.3**  $f: (X, \tau_1, \tau_2) \rightarrow (Z, \eta_1, \eta_2)$  is contra  $(i, j) - S_C -$  continuous.

**Theorem 3.12:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous at  $x \in X$  if and only if for each  $j -$  closed set  $V$  containing  $f(x)$ , there exists an  $(i, j) - S_C -$  open set  $U$  containing  $x$  such that  $f(U) \subseteq V$ .

**Proof:** Follows from **Theorem 3.4**.

**Theorem 3.13:** If a function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous at  $x$ , then for each  $j -$  closed set  $A$  containing  $f(x)$ , there exists an  $i -$  closed set  $F$  such that

$$f(F) \subseteq A.$$

**Proof:** Let  $A$  be any  $j -$  closed set containing  $f(x)$ . Since  $f$  is contra  $(i, j) - S_C -$  continuous, so by **Theorem 3.12** there exists an  $(i, j) - S_C -$  open set  $U$  containing  $x$  such that  $f(U) \subseteq A$ . Since  $U$  is  $(i, j) - S_C -$  open set, so for all  $x \in U$ , there exists an  $i -$  closed set  $F$  such that

$x \in F \subseteq U$ , this implies that  $f(F) \subseteq f(U) \subseteq A$ . Hence,  $f(F) \subseteq A$ .

**Theorem 3.14:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C -$  continuous if and only if  $f$  is  $j -$  contra semi - continuous and for each  $x \in X$  and each  $j -$  closed set  $V$  of  $Y$  containing  $f(x)$ , there exists an  $i -$  closed set  $F$  in  $X$  containing  $x$  such that  $f(F) \subseteq V$ .

**Proof:**

**If part:** Let  $x \in X$  and let  $V$  be any  $j$  – closed set of  $Y$  containing  $f(x)$ . Since  $f$  is contra  $(i, j) - S_C$  – continuous, so by **Theorem 3.4** there exists an  $(i, j) - S_C$  – open set  $U$  in  $X$  containing  $x$  such that  $f(U) \subseteq V$ . Since  $U$  is  $(i, j) - S_C$  – open set, then for each  $x \in U$ , there exists an  $i$  – closed set  $F$  such that  $x \in F \subseteq U$ . Therefore,  $f(F) \subseteq V$  and since  $f$  is contra  $(i, j) - S_C$  – continuous, so  $f$  is  $j$  – contra semi – continuous.

**Only if part:** Let  $V$  be any  $j$  – closed set in  $Y$ . To show  $f^{-1}(V)$  is  $(i, j) - S_C$  – open set in  $X$ .

Since,  $f$  is  $j$  – contra semi – continuous, then  $f^{-1}(V)$  is  $j$  – semi – open set in  $X$ . Let

$x \in f^{-1}(V)$ , then  $f(x) \in V$ , by hypothesis there exist an  $i$  – closed set  $F$  in  $X$  containing  $x$  such that  $f(F) \subseteq V$ , implies that  $x \in F \subseteq f^{-1}(V)$ . Therefore,  $f^{-1}(V)$  is  $(i, j) - S_C$  – open set in  $X$ . Hence, by **Corollary 3.3**  $f$  is contra  $(i, j) - S_C$  – continuous.

**Theorem 3.15:** Let  $f_1: (X_1, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  and  $f_2: (X_2, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  be two contra  $(i, j) - S_C$  – continuous functions, where  $(Y, \sigma_j)$  is Urysohn space. Then

$\{(x_1, x_2): f_1(x_1) = f_2(x_2)\}$  is  $(i, j) - S_C$  – closed set in the product space  $X_1 \times X_2$ .

**Proof:** Let  $A = \{(x_1, x_2): f_1(x_1) = f_2(x_2)\}$ . To show  $(X_1 \times X_2) \setminus A$  is  $(i, j) - S_C$  – open set.

Let  $(x_1, x_2) \notin A$ . Then  $f_1(x_1) \neq f_2(x_2)$ , since  $(Y, \sigma_j)$  is Urysohn space, then there exist  $j$  – open sets  $V_1$  and  $V_2$  containing  $f_1(x_1)$  and  $f_2(x_2)$  respectively, such that  $j - Cl(V_1) \cap j - Cl(V_2) = \emptyset$ .

Since,  $f_k$  ( $k = 1, 2$ ) is contra  $(i, j) - S_C$  – continuous,  $f_k^{-1}(j - Cl(V_k))$  is  $(i, j) - S_C$  – open set containing  $x_k$  in  $X_k$  ( $k = 1, 2$ ). Hence, by **Theorem 2.11**  $f_1^{-1}(j - Cl(V_1)) \times f_2^{-1}(j - Cl(V_2))$  is an  $(i, j) - S_C$  – open set. Further  $(x_1, x_2) \in f_1^{-1}(j - Cl(V_1)) \times f_2^{-1}(j - Cl(V_2)) \subseteq (X_1 \times X_2) \setminus A$ . Therefore  $(X_1 \times X_2) \setminus A$  is  $(i, j) - S_C$  – open set. Hence,  $A$  is  $(i, j) - S_C$  – closed set in the product space  $X_1 \times X_2$ .

**Corollary 3.16:** If a function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C$  – continuous and  $(Y, \sigma_j)$  is Urysohn space, then  $A = \{(x_1, x_2): f(x_1) = f(x_2)\}$  is  $(i, j) - S_C$  – closed set in the product space  $X \times X$ .

**Proof:** Follows from **Theorem 3.15**.

**Theorem 3.17:** A function  $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$  is contra  $(i, j) - S_C$  – continuous, if for each  $x \in X$ , there exist an  $i$  – regular closed and  $j$  – regular closed set  $A$  of  $X$  containing  $x$  such that  $f|_A: A \rightarrow Y$  is contra  $(i, j) - S_C$  – continuous.

**Proof:** Let  $F$  be any  $j$  – closed set of  $Y$  containing  $f(x)$ . Since  $f|_A$  is contra  $(i, j) - S_C$  – continuous, so there exists an  $(i, j) - S_C$  – open set  $U$  in  $A$  containing  $x$  such that  $(f|_A)(U) \subseteq F$ . Since  $A \in i - RO(X) \cap j - RO(X)$ , so by **Theorem 2.12**  $U$  is  $(i, j) - S_C$  – open set in  $X$  and hence,  $f(U) \subseteq F$ . Therefore, by **Theorem 3.12**  $f$  is contra  $(i, j) - S_C$  – continuous.

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