Journal homepage www.jzs.uos.edu.krd
Journal of Zankoi Sulaimani
Part-A- (Pure and Applied Sciences)

Application of the Local fractional Adomian Decomposition and Series Expansion Methods for Solving Telegraph Equation on Cantor Sets

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Article info

Original: 16 Nov. 2014
Revised: 26 Jan. 2015
Accepted: 10 Mar. 2015
Published online:
20 June 2015

Key Words:
Fractional telegraph equation; Local fractional Adomian decomposition method; Local fractional series expansion method; Local fractional operators.

Abstract

In this paper, we will compare between the local fractional Adomian decomposition method (LFADM) and local fractional series expansion method (LFSEM) for solving the telegraph equation on Cantor sets within local fractional operator. The results obtained by (LFADM) are compared with the results obtained by (LFSEM). Some examples are given to illustrate the efficiency and accuracy of the presented methods.

Introduction

In this present paper we consider the telegraph equations on Cantor sets involving local fractional derivatives

$$\frac{\partial^{2\alpha} u(x,y)}{\partial x^{2\alpha}} = k_1 \frac{\partial^{2\alpha} u(x,y)}{\partial y^{2\alpha}} + k_2 \frac{\partial^{\alpha} u(x,y)}{\partial y^{\alpha}} + k_3 u(x,y), \tag{1}$$

where k_1, k_2 and k_3 are the real constants and with fractal initial conditions

$$u(0,y) = \varphi(y),$$
$$\frac{\partial^{\alpha} u(0,y)}{\partial x^{\alpha}} = \psi(y), \tag{2}$$

where $\varphi(y)$ and $\psi(y)$ are continuous functions.

The telegraph equations are a pair of linear partial differential equations which describe the evolution of voltage and current on an electrical transmission line with distance and time. The telegraph equations are due to Oliver Heaviside who in the 1880s developed the transmission line model. This model demonstrates that the electromagnetic waves can be reflected on the wire, and that appear wave patterns along the transmission line. Recently, the local fractional decomposition method and local fractional series expansion method were applied to solve the partial differential equations arising in mathematical physics such as Laplace equation ***[1,2]***, heat equation ***[3]***, wave equation ***[4]***, Klein-Gordon equation ***[5]***, Helmholtz equation ***[6]***. In this paper, we investigate the application of local fractional Adomian decomposition method and local fractional series expansion method for solving telegraph equation on Cantor sets within local fractional operators.

Local Fractional Adomian Decomposition Method

We can written the equation (1) in the form:

$$L_{xx}^{(2\alpha)}u(x,y) = k_1 L_{yy}^{(2\alpha)}u(x,y) + k_2 L_y^{(\alpha)}u(x,y) + k_3 u(x,y) . \tag{3}$$

Applying the inverse operator $L_{xx}^{(-2\alpha)}$ to both sides of (3) yields

$$L_{xx}^{(-2\alpha)}\, L_{xx}^{(2\alpha)}u(x,y) = L_{xx}^{(-2\alpha)}\left(k_1 L_{yy}^{(2\alpha)}u + k_2 L_y^{(\alpha)}u + k_3 u\right), \tag{4}$$

so that

$$u(x,y) = \varphi(y) + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\psi(y) + L_{xx}^{(-2\alpha)}\left(k_1 L_{yy}^{(2\alpha)}u + k_2 L_y^{(\alpha)}u + k_3 u\right). \tag{5}$$

Now, we decompose the unknown function $u(x,y)$ as a sum of components defined by the series:

$$u(x,y) = \sum_{n=0}^{\infty} u_n(x,y) .$$

(6)

Substituting (6) into (5), we have

$$\sum_{n=0}^{\infty} u_n = \varphi(y) + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\psi(y) + L_{xx}^{(-2\alpha)}\left(k_1 L_{yy}^{(\alpha)}\left(\sum_{n=0}^{\infty} u_n\right) + k_2 L_y^{(\alpha)}\left(\sum_{n=0}^{\infty} u_n\right) + k_3\left(\sum_{n=0}^{\infty} u_n\right)\right).$$

(7)

It is normal to define the recursive formula by

$$\begin{aligned} &u_0(x,y) = \varphi(y) + \frac{x^{\alpha}}{\Gamma(1+\alpha)}\psi(y), \\ &u_{n+1}(x,y) = L_{xx}^{(-2\alpha)}\left(k_1 L_{yy}^{(2\alpha)}u_n + k_2 L_y^{(\alpha)}u_n + k_3 u_n\right),\ n \ge 0. \end{aligned} \tag{8}$$

Local Fractional Series Expansion Method

We can written the equation (1) in the form:

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}} = L_{\alpha}u(x,y) , \tag{9}$$

where $L_{\alpha}u(x,y) = k_1 L_{yy}^{(2\alpha)}u(x,y) + k_2 L_y^{(\alpha)}u(x,y) + k_3 u(x,y)$ is linear local fractional derivative operator of order 2α with respect to y.

In accordance with the results in [4,5], there are multiterm separated functions of independent variables x and y reads as

$$u(x,y) = \sum_{i=0}^{\infty} T_i(x) U_i(y) , \tag{10}$$

where $T_i(x)$ and $U_i(y)$ are local fractional continuous functions.

Moreover, there is a nondifferential series term

$$T_i(x)=\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}, \tag{11}$$

so that

$$u(x,y)=\sum_{i=0}^{\infty}\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}U_i(y). \tag{12}$$

In view of (12), we obtain

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}=\sum_{i=0}^{\infty}\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}U_{i+2}(y),$$

$$L_\alpha u(x,y)=L_\alpha\left(\sum_{i=0}^{\infty}\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}U_i(y)\right)=\sum_{i=0}^{\infty}\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}\left(L_\alpha U_i\right)(y). \tag{13}$$

Making use of (13), we get

$$\sum_{i=0}^{\infty}\frac{1}{\Gamma(1+i\alpha)}x^{i\alpha}U_{i+2}(y)=\sum_{i=0}^{\infty}\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}\left(L_\alpha U_i\right)(y). \tag{14}$$

Hence, from (14), the recursion reads as follows:

$$U_{i+2}(y)=\left(L_\alpha U_i\right)(y). \tag{15}$$

By using (15), we arrive at the following result:

$$u(x,y)=\sum_{i=0}^{\infty}\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}U_i(y). \tag{16}$$

Illustrate Examples

In this section three examples for the telegraph equation on Cantor sets are presented in order to demonstrate the simplicity and the efficiency of the above methods.

Example 1. Let us consider the telegraph equation on Cantor sets:

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}=\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u(x,y)}{\partial y^{\alpha}}-u(x,y), \tag{17}$$

subject to the fractal initial conditions

$$u(0,y)=E_\alpha(-y^\alpha),\quad \frac{\partial^{\alpha}u(0,y)}{\partial x^{\alpha}}=E_\alpha(-y^\alpha). \tag{18}$$

(1) By using local fractional Adomian decomposition method.

Operating with $L_{xx}^{(-2\alpha)}$ on (17) and using (18) yields

$$u(x,y)=E_\alpha(-y^\alpha)+\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(-y^\alpha)+L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u}{\partial y^{\alpha}}-u\right). \tag{19}$$

Substituting (6) into (19), we get

$$\sum_{n=0}^{\infty}u_n(x,y)=E_\alpha(-y^\alpha)+\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(-y^\alpha)+L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}}{\partial y^{2\alpha}}\left(\sum_{n=0}^{\infty}u_n\right)+\frac{\partial^{\alpha}}{\partial y^{\alpha}}\left(\sum_{n=0}^{\infty}u_n\right)-\sum_{n=0}^{\infty}u_n\right). \tag{20}$$

The local fractional Adomian decomposition method suggests the relation

$$\begin{aligned} u_0(x,y)&=E_\alpha(-y^\alpha)+\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(-y^\alpha)\\ u_{n+1}(x,y)&=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_n}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u_n}{\partial y^{\alpha}}-u_n\right),n\geq 0, \end{aligned} \tag{21}$$

where the components of the solution $u(x,y)$ given by

$$u_0(x,y)=E_\alpha(-y^\alpha)+\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(-y^\alpha)\,,$$

$$u_1(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_0}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u_0}{\partial y^{\alpha}}-u_0\right)=-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}E_\alpha(-y^\alpha)-\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha(-y^\alpha)\,,$$

$$u_2(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_1}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u_1}{\partial y^{\alpha}}-u_1\right)=\frac{x^{4\alpha}}{\Gamma(1+4\alpha)}E_\alpha(-y^\alpha)+\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}E_\alpha(-y^\alpha)\,, \tag{22}$$

$$u_3(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_2}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u_2}{\partial y^{\alpha}}-u_2\right)=-\frac{x^{6\alpha}}{\Gamma(1+6\alpha)}E_\alpha(-y^\alpha)-\frac{x^{7\alpha}}{\Gamma(1+7\alpha)}E_\alpha(-y^\alpha)\,,$$

follow immediately. In view of (22), the solution in a series form is given by

$$\begin{aligned}u(x,y)&=E_\alpha(-y^\alpha)\left(1-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{x^{4\alpha}}{\Gamma(1+4\alpha)}-\cdots\right)+E_\alpha(-y^\alpha)\left(\frac{x^{\alpha}}{\Gamma(1+\alpha)}-\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}-\cdots\right)\\&=E_\alpha(-y^\alpha)\cos_\alpha(x^\alpha)+E_\alpha(-y^\alpha)\sin_\alpha(x^\alpha)\,.\end{aligned} \tag{23}$$

(2) By using local fractional expansion series
Following (15), we have recursive formula

$$\begin{aligned}U_{i+2}(y)&=\left(\frac{\partial^{2\alpha}U_i}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}U_i}{\partial y^{\alpha}}-u_i\right)(y),\\U_0(y)&=E_\alpha(-y^\alpha),\\U_1(y)&=E_\alpha(-y^\alpha),\end{aligned} \tag{24}$$

Hence, we get

$$\begin{aligned}U_0(y)&=E_\alpha(-y^\alpha),\\U_1(y)&=E_\alpha(-y^\alpha),\\U_2(y)&=\left(\frac{\partial^{2\alpha}U_0}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}U_0}{\partial y^{\alpha}}-u_0\right)(y)=-E_\alpha(-y^\alpha),\\U_3(y)&=\left(\frac{\partial^{2\alpha}U_1}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}U_1}{\partial y^{\alpha}}-U_1\right)(y)=-E_\alpha(-y^\alpha),\end{aligned} \tag{25}$$

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and so on.
Therefore, through (25) we get the solution

$$\begin{aligned}u(x,y)&=\sum_{i=0}^{\infty}\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}U_i(y)\\&=E_\alpha(-y^\alpha)\left(1-\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{x^{4\alpha}}{\Gamma(1+4\alpha)}-\cdots\right)+E_\alpha(-y^\alpha)\left(\frac{x^{\alpha}}{\Gamma(1+\alpha)}-\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}-\cdots\right)\\&=E_\alpha(-y^\alpha)\cos_\alpha(x^\alpha)+E_\alpha(-y^\alpha)\sin_\alpha(x^\alpha)\,.\end{aligned} \tag{26}$$

From Eqs. (23) and (26), the approximate solution of the given problem (17) by using local fractional Adomian decomposition method is the same results as that obtained by the local fractional series expansion method and it clearly appears that the approximate solution remains closed form to exact solution.

Example 2. Let us consider the telegraph equation on Cantor sets:

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}=\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}u(x,y)}{\partial y^{\alpha}}+u(x,y)\,, \tag{27}$$

subject to the fractal initial conditions

$$u(0,y)=1-E_{\alpha}(-y^{\alpha})\,,\quad \frac{\partial^{\alpha}u(0,y)}{\partial x^{\alpha}}=0. \tag{28}$$

(1) By using local fractional Adomian decomposition method.
Operating with $L_{xx}^{(-2\alpha)}$ on (27) and using (28) yields

$$u(x,y)=1-E_{\alpha}(-y^{\alpha})+L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}u}{\partial y^{\alpha}}+u\right). \tag{29}$$

Substituting (6) into (29), we get

$$\sum_{n=0}^{\infty}u_n(x,y)=1-E_{\alpha}(-y^{\alpha})+L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}}{\partial y^{2\alpha}}\left(\sum_{n=0}^{\infty}u_n\right)+2\frac{\partial^{\alpha}}{\partial y^{\alpha}}\left(\sum_{n=0}^{\infty}u_n\right)+\sum_{n=0}^{\infty}u_n\right). \tag{30}$$

The local fractional Adomian decomposition method suggests the relation

$$\begin{aligned}&u_0(x,y)=1-E_{\alpha}(-y^{\alpha})\\&u_{n+1}(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_n}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}u_n}{\partial y^{\alpha}}+u_n\right),n\ge 0,\end{aligned} \tag{31}$$

where the components of the solution $u(x,y)$ given by

$$\begin{aligned}&u_0(x,y)=1-E_{\alpha}(-y^{\alpha}),\\&u_1(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_0}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}u_0}{\partial y^{\alpha}}+u_0\right)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)},\\&u_2(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_1}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}u_1}{\partial y^{\alpha}}+u_1\right)=\frac{x^{4\alpha}}{\Gamma(1+4\alpha)},\\&u_3(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_2}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}u_2}{\partial y^{\alpha}}+u_2\right)=\frac{x^{6\alpha}}{\Gamma(1+6\alpha)},\end{aligned} \tag{32}$$

follow immediately. In view of (32), the solution in a series form is given by

$$u(x,y)=\cosh_{\alpha}(x^{\alpha})-E_{\alpha}(y^{\alpha})\,.$$

(33)

(2) By using local fractional expansion series
Following (15), we have recursive formula

$$\begin{aligned}&U_{i+2}(y)=\left(\frac{\partial^{2\alpha}U_i}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}U_i}{\partial y^{\alpha}}+U_i\right)(y),\\&U_0(y)\ =1-E_{\alpha}(-y^{\alpha}),\\&U_1(y)\ =0,\end{aligned} \tag{34}$$

Hence, we get

$$\begin{aligned}&U_0(y)=1-E_{\alpha}(-y^{\alpha}),\\&U_1(y)=0,\\&U_2(y)=\left(\frac{\partial^{2\alpha}U_0}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}U_0}{\partial y^{\alpha}}+U_0\right)(y)=1,\end{aligned}$$

$$U_3(y)=\left(\frac{\partial^{2\alpha}U_1}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}U_1}{\partial y^{\alpha}}+U_1\right)(y)=0, \tag{35}$$

$$U_4(y)=\left(\frac{\partial^{2\alpha}U_2}{\partial y^{2\alpha}}+2\frac{\partial^{\alpha}U_2}{\partial y^{\alpha}}+U_2\right)(y)=1,$$

and so on.

Therefore, through (35) we get the solution

$$u(x,y)=\sum_{i=0}^{\infty}\frac{x^{i\alpha}}{\Gamma(1+i\alpha)}U_i(y)$$

$$=\left(1+\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{x^{4\alpha}}{\Gamma(1+4\alpha)}+\frac{x^{6\alpha}}{\Gamma(1+6\alpha)}+\cdots\right)-E_\alpha(-y^\alpha)$$

$$=\cosh_\alpha(x^\alpha)-E_\alpha(y^\alpha). \tag{36}$$

From Eqs. (33) and (36), the approximate solution of the given problem (27) by using local fractional Adomian decomposition method is the same results as that obtained by the local fractional series expansion method and it clearly appears that the approximate solution remains closed form to exact solution.

Example 3. Let us consider the telegraph equation on Cantor sets:

$$\frac{\partial^{2\alpha}u(x,y)}{\partial x^{2\alpha}}=\frac{\partial^{2\alpha}u(x,y)}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u(x,y)}{\partial y^{\alpha}}+u(x,y)$$

(37)

subject to the fractal initial conditions

$$u(0,y)=E_\alpha(-y^\alpha)\ ,\quad \frac{\partial^{\alpha}u(0,y)}{\partial x^{\alpha}}=E_\alpha(-y^\alpha). \tag{38}$$

(1) By using local fractional Adomian decomposition method.

The local fractional Adomian decomposition method suggests the relation

$$u_0(x,y)=E_\alpha(-y^\alpha)+\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(-y^\alpha)$$

$$u_{n+1}(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_n}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u_n}{\partial y^{\alpha}}+u_n\right),n\ge 0, \tag{39}$$

where the components of the solution $u(x,y)$ given by

$$u_0(x,y)=E_\alpha(-y^\alpha)+\frac{x^\alpha}{\Gamma(1+\alpha)}E_\alpha(-y^\alpha)\ ,$$

$$u_1(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_0}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u_0}{\partial y^{\alpha}}+u_0\right)=\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}E_\alpha(-y^\alpha)+\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha(-y^\alpha)\ ,$$

$$u_2(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_1}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u_1}{\partial y^{\alpha}}-u_1\right)=\frac{x^{4\alpha}}{\Gamma(1+4\alpha)}E_\alpha(-y^\alpha)+\frac{x^{5\alpha}}{\Gamma(1+5\alpha)}E_\alpha(-y^\alpha)\ ,$$

(40)

$$u_3(x,y)=L_{xx}^{(-2\alpha)}\left(\frac{\partial^{2\alpha}u_2}{\partial y^{2\alpha}}+\frac{\partial^{\alpha}u_2}{\partial y^{\alpha}}-u_2\right)=\frac{x^{6\alpha}}{\Gamma(1+6\alpha)}E_\alpha(-y^\alpha)+\frac{x^{7\alpha}}{\Gamma(1+7\alpha)}E_\alpha(-y^\alpha)\ ,$$

follow immediately. In view of (40), the solution in a series form is given by

$$u(x,y)=E_\alpha(-y^\alpha)\left(1+\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\cdots\right)$$

$$= E_\alpha(-y^\alpha)E_\alpha(x^\alpha) = E_\alpha(x^\alpha - y^\alpha)\,. \tag{41}$$

(2) By using local fractional expansion series
Following (15), we have recursive formula

$$\begin{aligned} U_{i+2}(y) &= \left(\frac{\partial^{2\alpha}U_i}{\partial y^{2\alpha}} + \frac{\partial^{\alpha}U_i}{\partial y^{\alpha}} + U_i\right)(y), \\ U_0(y) &= E_\alpha(-y^\alpha), \\ U_1(y) &= E_\alpha(-y^\alpha), \end{aligned} \tag{42}$$

Hence, we get

$$\begin{aligned} U_0(y) &= E_\alpha(-y^\alpha), \\ U_1(y) &= E_\alpha(-y^\alpha), \\ U_2(y) &= \left(\frac{\partial^{2\alpha}U_0}{\partial y^{2\alpha}} + \frac{\partial^{\alpha}U_0}{\partial y^{\alpha}} + U_0\right)(y) = E_\alpha(-y^\alpha), \\ U_3(y) &= \left(\frac{\partial^{2\alpha}U_1}{\partial y^{2\alpha}} + \frac{\partial^{\alpha}U_1}{\partial y^{\alpha}} + U_1\right)(y) = E_\alpha(-y^\alpha), \end{aligned} \tag{43}$$

and so on.
Therefore, through (43) we get the solution

$$\begin{aligned} u(x,y) &= \sum_{i=0}^{\infty} \frac{x^{i\alpha}}{\Gamma(1+i\alpha)} U_i(y) \\ &= E_\alpha(-y^\alpha)\left(1 + \frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)} + \cdots\right) \\ &= E_\alpha(-y^\alpha)E_\alpha(x^\alpha) = E_\alpha(x^\alpha - y^\alpha)\,. \end{aligned} \tag{44}$$

From (41) and (44), the approximate solution of the given problem (37) by using local fractional Adomian decomposition method is the same results as that obtained by the local fractional series expansion method and it clearly appears that the approximate solution remains closed form to exact solution.

Conclusions

The main goal of this work is to conduct a comparative study between the local fractional Adomian decomposition method and local fractional series expansion method. The two methods are demonstrated as an effective method for solutions of a wide class of problems. Analytical solutions of the telegraph equation on Cantor sets involving local fractional derivatives are successfully developed by recurrence relations resulting in convergent series solutions. The suggested methods are potential tool for development of approximate solutions of local fractional partial differential equations with fractal initial value conditions, which, of course, draws new problems beyond the scope of the present work.

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