



## Moment Capacity Charts for Strengthening of RC Beams With Near Surface Mounted FRP Reinforcement

*Professor Dr. Mohamad R. Abdulkadir and Avin H. Abdullah MSc*

*Departement of Civil Engineerng, Faculty of Engineering, University of Sulaimani,  
 E mail: [raoufkadir@hotmail.com](mailto:raoufkadir@hotmail.com) , [avin\\_hasib2009@hotmail.com](mailto:avin_hasib2009@hotmail.com)*

### Article info

Original: 11.11.2015  
 Revised:05.01.2016  
 Accepted:30.01.2016  
 Published online:  
 20.06.2016

#### Key Words:

*Beams,  
 Near-surface  
 mounted,  
 fibre reinforced  
 polymer,  
 moment capacity*

### Abstract

Near surface- mounted fibre reinforced polymer is a new technique for strengthening structural members beside externally applied fibre reinforced polymer (FRP) technology. In NSM technique, the FRP material is applied to the tension region of the member inside pre-prepared grooves. In externally applied FRP the debonding of the fibres is a major issue thus, the full utilisation of FRP material can not be achieved, and the fibres are more susceptible to environmental effects. With NSM FRP, these issues can be controlled more effectively. ACI Committee 440 [1] has presented guidelines for design and construction of externally FRP method for strengthening of concrete structures. Although many experiments have been conducted on NSM FRP in flexure strengthening of RC beams, no code is introduced yet. Theoretical studies to derive equations for calculating moment capacity of strengthened beams have been introduced but mostly require several trial procedures to reach the required effective strain for FRP. In this study, a moment capacity chart is introduced based on several assumptions in the derivation of flexure capacity equation. To avoid the brittle behaviour of FRP, it is suggested that the utilization of FRP to be limited to 70% of its ultimate capacity for CFRP, and an equivalent area of reinforcement is adopted for simplicity. Experimental results of other researchers have been verified with the proposed charts, and a good comparison of results have being achieved.

### Introduction

Strengthening of reinforced concrete members such as beams and columns solves many problems of engineering structures such as extra loads that might be imposed on the structures, any imperfection in the design process, any changes in building function, and issues associate with the building materials. Several methods for strengthening have been discovered worldwide in innovation and rehabilitation field. Near surface-mounted fibre reinforced polymer is a new technique for strengthening which received popularity beside externally applied fibre reinforced polymer (FRP) technology. The idea of NSM was first implemented in 1949 in strengthening a bridge in Sweden using embedded steel. However, due to the heavy weight of steel and susceptibility to corrosion and other environmental issues, it was replaced with fibre reinforced polymer that is regarded as a suitable material for strengthening. Externally bonded FRP technique was firstly developed, and ACI Committee 440 presented a guideline for design and construction of externally FRP method for strengthening of concrete structures [1].

In externally FRP technology, seven types of failure were observed in which bond issue take the precedence [2]. Thus, the full utilisation of FRP material could not be achieved. As a result, attempts were not finalised, and a new strengthening scheme has been introduced which is NSM FRP technology since the early of 2000. It has become an alternative for other strengthening methods due to its great advantages such as good resistance to environmental issues and fewer bond problems [3] [4]. In NSM technique, the FRP material applies to the tension region of the selected beam inside pre-prepared grooves. According to De Lorenzis, et al, El-Hacha, and Rizkalla, and Jung, et al [5-7], the efficiency of NSM FRP in improving the capacity of the strengthened beams were confirmed. To obtain the required flexure strengthening capacity, the bond should be controlled in the design process and during the process of strengthening. As far as the practical process of strengthening concerned. Firstly, special care should be given to the internal steel reinforcement of the strengthened structural member during grooving process to avoid any damage. Secondly, cleaning the grooves from any fine particles is important to provide a proper bond between concrete and bonding agent, and any imperfection within the structural element such as cracks should be repaired before strengthening. Finally, the surface of the grooves after inserting FRP materials and adhesive should be made plain and levelled to avoid any concentration of stress, and proper curing of the adhesive should not be forgotten [8].

During design process selection of different parameters has a great influence on controlling bond issues such as groove size, bond length, surface texture and types of FRP bars are found to have an influence on bond failure. Delorenzis [9] concluded through a bond test with square dimensions that to prevent debonding of FRP materials, the value of groove width ( $b_g$ ) should range from (1.5 to 2) of the diameter of FRP bars ( $d_b$ ). Sharaky, et al [10] Pointed out that debonding failure type was delayed when groove size increased (from 1.5 to 2)  $d_b$ . This shows an agreement between the two previous tests. Hassan and Rizkalla[11] concluded through a test that clear spacing between the grooves should range from (0.2 to 0.25)  $d_b$  and edge distance to be limited to 4  $d_b$ .

## **Research Significance**

NSM FRP reinforcement is regarded as one of the most significant methods of strengthening concrete structures and a valid alternative to externally FRP technology in enhancing the capacity of concrete members such as beams and columns as well as masonry structures. Although many experiments have been conducted on NSM FRP in flexure strengthening of RC beams, no code is introduced yet. Theoretical studies to derive an equation for calculating the capacity of strengthened beam have been introduced but mostly require several trial procedures to reach the required effective strain for FRP.

In this study, a moment capacity chart is introduced based on several assumptions in the derivation of flexure capacity equation and balanced reinforcement ratio. FRP is an elastic material up to failure when subjected to tensile stress. Thus, this can be regarded as the only brittle behaviour of FRP. To avoid the brittle behaviour, it is suggested in this research that the utilization of FRP strength to be limited to 70% of its ultimate capacity, and an equivalent area of reinforcement is adopted for simplicity.

## **Flexural Strengthening**

A theoretical model was introduced by Yost, et al [4] identifying the two flexural failure modes of concrete crushing and rupture of FRP strips based on comparison with balanced condition and derivation of the nominal capacity of the strengthened beams ( $M_n$ ). The author suggested that the theoretical analysis can replace the trial and error step that was offered by ACI Committee 440 [1] for externally applied FRP. Calculation of ultimate moment and FRP strain were derived analytically by Al-Mahmoud, et al [12]. An experimental and analytical study on RC beams strengthened with NSM FRP, and steel bars was carried out by Almusallam et al [13], since debonding of FRP was not observed in the test, perfect bond was assumed in

calculating flexural capacity of the strengthened beams at balanced condition through a detailed procedure based on both codes ACI Committee 318 [14] and ACI Committee 440 [15]. A new parameter named FRP reinforcement coefficient ( $\alpha_r$ ) was introduced as a result of the model study. It was suggested that the minimum value of ( $\alpha_r$ ) to be used in design should not be less than 30 which correspond to a utilisation of 90% of the ultimate full capacity of FRP.

A data base for almost all tests on NSM FRP technology on flexure strengthening of RC beams was introduced by Abdullah [16]. Moreover, to avoid the tedious trial and error procedure or deriving long equations to calculate flexural strengthening capacity of RC members, the author derived an analytical model using a Matlab software based on the fundamental law of materials and several assumptions of ACI Committee 440 [1]. Design philosophy and approach for the proper use of externally applied FRP strengthening materials were proposed by ACI Committee 440[1], whereas, no code is introduced by ACI Committee for NSM FRP strengthening technique. However, several design guide lines can be found in the literature that present the design philosophy for flexure and shear design of rectangular sections [17], [18]. Those depended mainly on several assumptions and principles, such as equilibrium state, plain sections remain plain, the linearity of FRP to failure and a full bond between FRP materials and surrounding concrete. A bond dependent coefficient that was named ( $K_m$ ) was another significant factor which proposed by Parretti and Nanni [18], where  $K_m$  according to test results ranged from 0.6 to 0.84 to prevent delamination of FRP bars/strips. This came in agreement with test results of Delorenzis and Nanni [19] and the approach adopted by ACI Committee 440 [1], in which a limitation factor for strain in EFRP was introduced. Besides, a strength reduction factor ( $\Phi$ ) was also suggested by ACI Committee 440 [1], in design process depending on the ductility performance.

$$\phi = \left\{ \begin{array}{l} 0.9 \quad \text{for } \varepsilon_s \geq 0.005 \\ 0.7 + \frac{0.2(\varepsilon_s - \varepsilon_y)}{0.005 - \varepsilon_y} \\ 0.7 \quad \text{for } \varepsilon_s \leq \varepsilon_y \end{array} \right. \quad \left| \quad \text{for } \varepsilon_y < \varepsilon_s < 0.005 \right.$$

### **Moment Capacity Chart**

An analytical model is proposed based on strain and stress distributions shown in figure 1 which consists of a rectangular section reinforced with a steel bar and strengthened with near surface longitudinal FRP bar bonded to the tension region of the member. Also both strain and stress distributions for the mentioned model are shown. The same design philosophy and assumptions that were explained in flexural strengthening section of this paper for previous literature on NSM FRP strengthening are adopted. Ultimate compressive strain in concrete is assumed to be 0,003.

Figure 2 represents the elastic behaviour of CFRP and the elastic-plastic behaviour of steel, which were modelled by using Excel sheet based on data extracted from Abdullah, [16] database. The average value of the ultimate strain of FRP, which were taken from the tests, is calculated. It is assumed that at the balanced condition when steel already have reached yielding stage FRP still propagating and approximately at the point where the strain of steel is (0.01) strain of CFRP reaches up to 70% of its ultimate strain. This can be adopted as a good platform for the derivation of the required equations, which are attached in Appendix A, to model the design chart.

A new term that is an equivalent depth of reinforcement ( $d_{eq}$ ) is suggested as well as equivalent reinforcement ratio ( $\rho_{eq}$ ) depending on ( $d_{eq}$ ) which is calculated by the equivalent area of FRP and steel reinforcement. After the model is produced, the equations are derived based on above assumptions as shown in Appendix A.

The equations of equivalent reinforcement ratio ( $\rho_{eq}$ ) and moment capacity factor or strength factor ( $R$ ), =

$\frac{M}{bd_{eq}^2}$  are as follows:

$$R = 0.9 \rho_{eq} f_y$$

$$\rho_{eq} = \frac{(A_s + 0.7 A_f \frac{f_{fu}}{f_y})}{bd_{eq}}$$

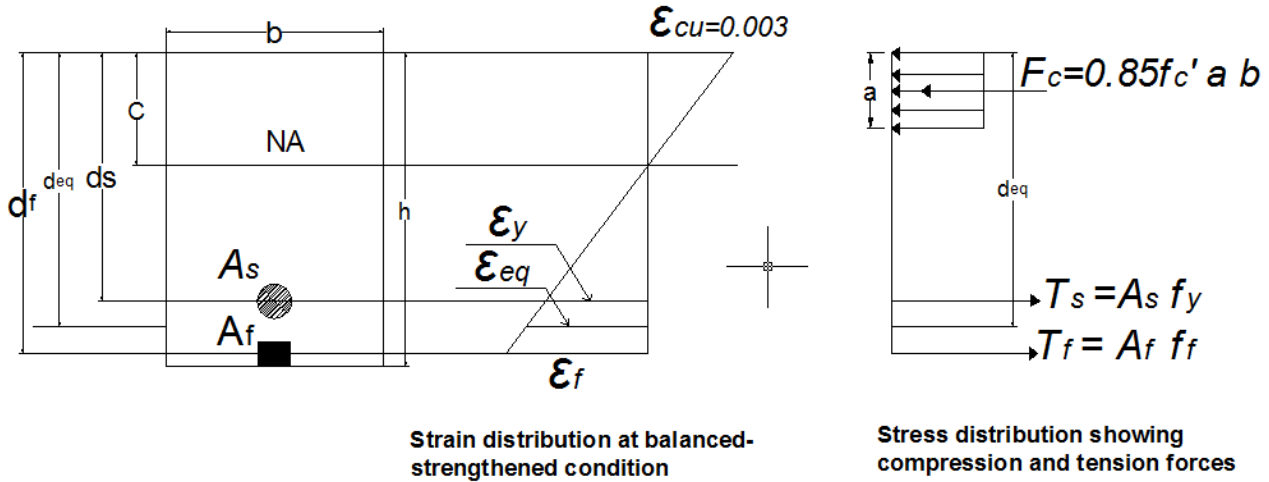


Fig. 1 Stress-strain distribution for RC rectangular section strengthened in flexure with NSMFRP

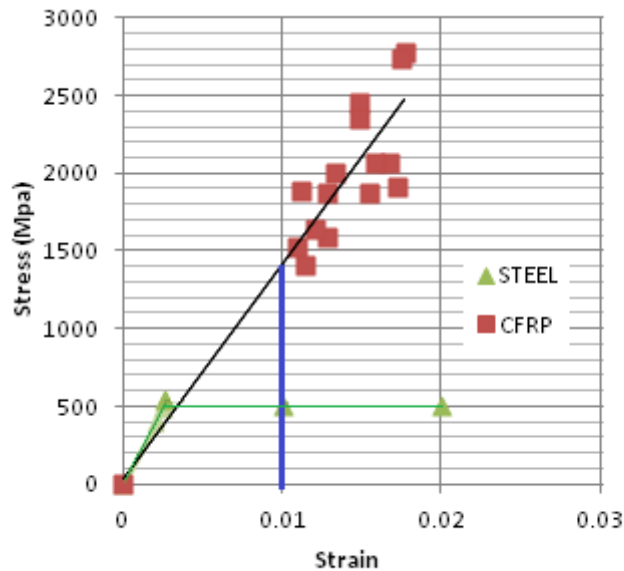


Fig 2 Idealized curve of CFRP and steel at yielding and ultimate condition

Based on assumptions that are presented, the moment capacity factor can be derived, and the chart can be drawn in relation to equivalent reinforcement ratio ( $\rho_{eq}$ ), Similar to charts used in the design of RC beams in the form of a set of lines for different ( $f_y$ ).

**Chart Verification and Discussion**

The design charts are drawn based on the equations derived in Appendix ( A ) and shown in figure (3). The charts are drawn for different longitudinal steel yield strengths. Previous experimental test results based on the database that was gathered by Abdullah [16] on previous flexure experiments on NSM CFRP are applied on the charts. The database contains a large number of flexure tests with different modes of failure such as concrete crushing, rupture of FRP and debonding failure modes. As long as the perfect bond was assumed in performing the design chart, specimens with debonding failure mode were excluded from Abdullah [16] database, only flexure failure types were considered as listed in table1 in Appendix ( C ). In order to apply the tests results on the chart, from the database table the followings were calculated: equivalent depth and equivalent reinforcement ratio  $d_{eq}, \rho_{eq}$  and actual moment capacity factor R that is equal to:  $R = M_{exp}/bd_{eq}^2$  Where ( $M_{exp}$ ) is the actual moment from the test results at ultimate limit state. Test results from Appendix C for CFRP samples with flexure failure modes are applied on the design charts and presented in Figs 3.

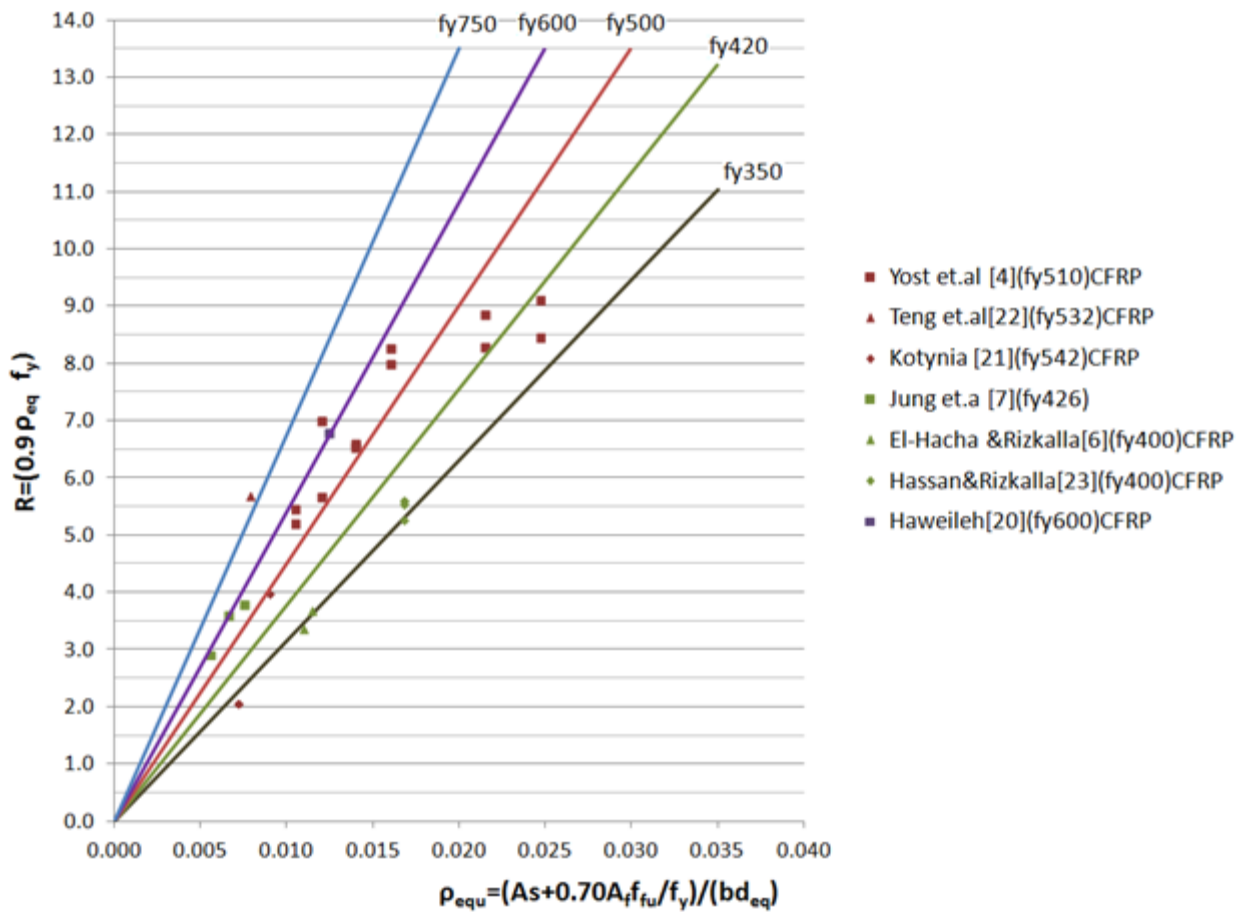


Fig 3 Design chart with application of test data

The test results on the charts show close agreement with the corresponding steel yield strength charts. For better comparison of the test results with the proposed charts, Table 2 was drawn which presents the numerical results of the tests from the data base with the ratio of estimated moment capacity to the actual test moment capacity. The ratios vary from 0.66 to 1.29 however the majority are within 20% above or below the

test value, and the overall ratio for all the tests is 1.02, indicating the estimated values equally overestimates the results for some and underestimates for others. To show the relation of the estimated moment capacities with the FRP strain attained in the actual tests at ultimate load, and if the discrepancies could be related to the actual strain attained as based on the calculation carried out by Abdulla [16]. Table 3 shows the ratio of the effective strain of FRP at the failure to the ultimate strain of the FRP used in the tests; the effective strains are calculated from Matlab program result of Abdullah [16] for the same tests. Those points, which are falling on the lines in the chart with moment ratios close to one, are found to be of the specimens that have approximately strain range around 70% of the ultimate state, whereas other specimens have more or fewer strain levels than 70%. This can be a good indication of the accuracy of the proposed design chart that can be a step forward to NSM FRP technology and contribute in further developing stages. The proposed charts can also be used for design purposes following the procedure presented in Appendix B. A reduction factor  $\phi = 0.9$  as proposed by ACI Committee 440 [1] should be applied. To further provide safety against brittleness of the FRP it is recommended to apply another reduction factor depending on the fibre type from 0.8 to 0.9 which may be called fibre reduction factor.

It is to be noted that the equations or the charts can be applied to ordinary reinforced concrete without strengthening simply by substituting zero for  $A_f$  area of FRP reinforcement that would turn into ordinary design equations with the assumption of moment arm being equal to  $0.9d$  which is a good approximation for most of the practical designs. Alternatively, it can be applied to beams reinforced only with inner FRP reinforcement in the same way substituting zero for  $A_s$  area of steel reinforcement.

Table 2 Comparison between experimental and estimated moments

Reference *	NO.	Beam notation	Types of FRP	$M_{exp}$ (KN.mm)	$M_{est}$ (KN.mm)	$M_{est}/M_{exp}$
Hawileh 2012	1	SC12 (VC30)	CFRP-bars	65400	64863.0	0.99
Kotynia 2012	2	NIISB/40/80	CFRP-strip	78000	86212.4	1.11
Yost et al. 2007	3	6-1Fa	CFRP-strip	30267.77	32375.9	1.07
	4	6-1Fb	CFRP-strip	28329.56	32375.9	1.14
	5	6-2Fa	CFRP-strip	30462.81	39228.3	1.29
	6	6-2Fb	CFRP-strip	32839.86	39228.3	1.19
	7	9-1Fa	CFRP-strip	34400.18	33432.6	0.97
	8	9-1Fb	CFRP-strip	34046.67	33432.6	0.98
	9	9-2Fa	CFRP-strip	45163.95	40285.0	0.89
	10	9-2Fb	CFRP-strip	43664.58	40285.0	0.92
	11	12-1Fa	CFRP-strip	36070.21	33432.6	0.93
	12	12-1Fb	CFRP-strip	37801.19	33432.6	0.88
	13	12-2Fa	CFRP-strip	41202.2	40285.0	0.98
	14	12-2Fb	CFRP-strip	50917.63	40285.0	0.79
Teng et al. 2006	15	B2900	CFRP-strip	59880	39762.4	0.66
Jung et al. 2005	16	NSM-PL-15	CFRP-strip	41207.25	30413.1	0.74
	17	ROD-MI-20	CFRP-rod	55755	42555.9	0.76
	18	PL-MI-20	CFRP-strip	51828	36470.0	0.70
El-Hacha and Rizkalla 2004	19	B2	CFRP-strip	62062	73402.4	1.18
	20	B3	CFRP-strip	68875	77275.2	1.12
Hassan and Rizkalla 2003	21	B5	CFRP-strip	49375	53940.2	1.09
	22	B6	CFRP-strip	46875	53940.2	1.15
	23	B7	CFRP-strip	50000	53940.2	1.08
	24	B8	CFRP-strip	50000	53940.2	1.08

Hawileh 2012=[20], Kotynia 2012=[21], Yost et al.2007=[4], Teng et al.2006=[22], Jung et al.2005=[7], El-Hacha and Rizkalla2004=[6],Hassan and Rizkalla 2003=[23]

Table 3 Effective strain of FRP at failure from Matlab program result of Abdullah [16]

Reference	NO.	Beam notation	$\epsilon_{fe}$	$\epsilon_{fe}/\epsilon_{fu}$
Hawileh 2012	1	SC12 (VC30)	0.0128425	1.0
Kotynia 2012	2	NISB/40/80	0.00288482	0.26
yost et al. 2007	3	6-1Fa	0.012118	1.0
	4	6-1Fb	0.005934	0.49
	5	6-2Fa	0.0046123	0.38
	6	6-2Fb	0.00578	0.48
	7	9-1Fa	0.01043	0.86
	8	9-1Fb	0.009988	0.82
	9	9-2Fa	0.00873	0.72
	10	9-2Fb	0.00873	0.72
	11	12-1Fa	0.011188	0.92
	12	12-1Fb	0.0121176	1.0
	13	12-2Fa	0.0090114	0.74
	14	12-2Fb	0.011391	0.94
Teng et al. 2006	15	B2900	0.0157863	1.0
Jung et al. 2005	16	NSM-PL-15	0.0148226	1.0
	17	ROD-MI-20	0.01546697	1.0
	18	PL-MI-20	0.01482265	1.0

## Conclusions

As a result of this study the following conclusions can be drawn:

1. Near surface mounted strengthening with FRP is an effective, promising new technique for enhancing the moment capacity of reinforced concrete members.
2. No provisions in ACI-code have yet been given for the design of this type of strengthening.

3. New design equations and charts have been proposed in this paper based on some simple approximations and assumptions to derive the moment capacity of the strengthened beams.
4. The proposed moment capacity charts are applied to previous experimental tests to estimate the moment carrying capacity and good agreement has been obtained.
5. The charts can be applied to both design and analysis of strengthened beams.

### **References:**

- [1] ACI440.IR-02 “*Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures*”, American concrete institute, Farmington Hills Michigan. (2002).
- [2] Fib TC. “*Externally bonded FRP reinforcement for RC structures*”, International Federation for structural concrete, Lausanne. (2001).
- [3] Delorenzis, L. and Nanni, A. “Bond between near-surface mounted fibre-reinforced polymer rods and concrete in structural strengthening”, *ACI-Structural Journal*, Vol. (99), No. S13, pp. 123-132. (2002).
- [4] Yost, J. R., Gross, S. P., Dinehart, D. W. and Mildenberg, J.J “*Flexural behaviour of concrete beams strengthened with near-surface-mounted CFRP strips*”. *ACI-Structural Journal*, Vol. (104), No. S41, pp. 430-437. (2007).
- [5] Delorenzis, L., Nanni, A. and Tegola, A. La “*Strengthening of Reinforced structures with near surface mounted FRP rods*”, (2000). [online] available from <scholar.google.co.uk> [2 July 2013]
- [6] El-Hacha, R. and Rizkalla, S. H. “*Near Surface Mounted Fibre Reinforced Polymer Reinforcements for Flexural Strengthening of Concrete Structures*”, *ACI-Structural Journal*, Vol. (101), No. S71, pp. 717-726. (2004).
- [7] Jung, W.,T., Park, Y.,H., Park, J., S., Kang, J., Y. and You, Y.,J. “*Experimental investigation on flexural behaviour of RC beams strengthened by NSM CFRP reinforcements*”, (2005). [online] available from <scholar.google.co.uk> [2 July 2013]
- [8] Badawi M, Soudki K. “*Flexural strengthening of RC beams with prestressed NSM CFRP rods – experimental and analytical investigation*”, *Construction and building materials*, Vol. (23)No. 10, pp. 292–300. (2009).
- [9] Delorenzis, L. “*Strengthening of RC structures with near surface mounted FRP rods*”, PhD Thesis, Department of Innovation Engineering, University of Lecce, Italy. (2002).
- [10] Sharaky, A., Torres, L., Beana, M. and Vilanova, I. “*Effect of different material and construction details on the bond behaviour of NSM FRP bars in concrete*”, *Construction and building materials*, Vol. (38), pp. 890-902. (2012)
- [11] Hassan, T. and Rizkalla, S. “*Bond mechanism of near-surface-mounted fibre –reinforced polymer bars for flexural strengthening of concrete structures*”, *ACI-Structural Journal*, Vol. (101), No. S82, pp. 830-839. (2004)
- [12] Al-Mahmoud, F., Castel, A., Francois, R. and Tournour, C. “*Strengthening of RC members with near-surface mounted CFRP rods*”, *Composite structures*”, Vol. (91), pp. 138-147. (2009).
- [13] Almusallam, T., H., Elsanadedy, H., M., Al-Salloum, Y., A. and Alsayed, S., H. “*Experimental and numerical investigation for the flexural strengthening of RC beams using near-surface mounted steel or GFRP bars*”, *Construction and building materials*, Vol. (40), pp.145-161. (2013).
- [14] ACI Committee 318 “*Building code requirements for structural concrete and commentary. ACI318-11*”, American concrete institute, Detroit, MI, USA. (2011).
- [15] ACI Committee 440 “*Guide for the Design and Construction of Structural Concrete reinforced with FRP bars. ACI440.IR-06-11*”, American concrete institute Detroit, MI, USA. (2006).
- [16] Abdullah, A., H., “*Flexural Strengthening of RC Members with Near Surface Mounted FRP Reinforcement*”, MSc Thesis, Civil Engineering Department, University of Nottingham, Nottingham. (2013).

- [17] Nanni, A. “North American design guidelines for concrete reinforcement and strengthening using FRP: principles, applications and unresolved issues”, Center for Infrastructure Engineering Studies, University of Missouri–Rolla, Rolla, USA. (2003).
- [18] Parretti, R. and Nanni, A. “Strengthening of RC Members Using Near-Surface Mounted FRP Composites: Design Overview”, Advances in Structural Engineering, Vol. (7), No. 5. (2004).
- [19] Delorenzis, L., Nanni, A. “Bond between Near-Surface Mounted FRP Rods and Concrete in Structural Strengthening”, ACI Structures Journal, Vol. (99), No. 2, pp. 123-133. (2002).
- [20] Hawileh, R., A. “Nonlinear finite element modelling of RC beams strengthened with NSM FRP rods”, Construction and building materials, Vol. (27), pp. 461-471. (2012).
- [21] Kotynia, R. “Bond between FRP and concrete in reinforced concrete beams strengthened with near surface mounted and externally bounded reinforcement”, Construction and building materials, Vol. (32), pp. 41-54. (2012).
- [22] Teng, J.G., Delorenzis, L., Wang, B., Li, R., Wong, T. N. and Lam, L. “Debonding failures of RC beams strengthened with near surface mounted CFRP strips”, Journal of composites for construction, Vol. (10), No. 2, pp. 92-105. (2006).
- [23] Hassan, T. and Rizkalla, S. “Investigation of bond in concrete structures strengthened with near surface mounted carbon fibre reinforced polymer strips”, Journal of composites for construction, Vol. (7), No. 3, pp. 248-257. (2003).

## APPENDIX A

### Derivation of Equations

#### SYMBOLS

$A_f$  = area of FRP (mm<sup>2</sup>) ,  $A_s$  = area of steel (mm<sup>2</sup>) ,  $f_c'$  = concrete compressive strength (MPa)

$b$  = width of beam (mm),  $d_f$  or  $d_{NSM}$  = distance from top fibre of beam to centre of FRP bar (mm)

$c_{beq}$  = depth of neutral axis at balanced condition (mm),  $f_{fu}$  = ultimate stress of FRP (MPa)

$f_f$  = actual stress of FRP (MPa),  $f_y$  = yield stress of internal reinforcement (MPa)

$\epsilon_y$  = yield strain of steel reinforcement,  $\epsilon_{cu}$  = crushing strain of concrete,  $\epsilon_{fu}$  = ultimate strain of FRP

$\epsilon_{fe}$  = effective strain of FRP ,  $E_f$  = modulus of elasticity of FRP (MPa)

$E_y$  = modulus of elasticity of steel (MPa)

From strain diagram of balanced failure condition in Fig 1 with  $\epsilon_{eq}$  being the strain at depth  $d_{eq}$  at which the steel has yielded ( $\epsilon_y$  about 0.01 ).

$$c_{beq} = \left( \frac{\epsilon_{cu}}{\epsilon_{eq} + \epsilon_{cu}} \right) d_{eq} , a_{beq} = \beta_1 c_{beq} = \beta_1 \left( \frac{\epsilon_{cu}}{\epsilon_{eq} + \epsilon_{cu}} \right) d_{eq}$$

Tension in steel =  $T_s = A_s f_y$  , Tension in fiber =  $T_f = A_f f_f$  .....  $f_f \leq f_{fu}$

In terms of equivalent area of steel  $T_f = A_f \frac{\epsilon_f}{\epsilon_y} f_y$

$$\Sigma T = A_s f_y + A_f \frac{\epsilon_f}{\epsilon_y} f_y, \quad \Sigma T = (A_s + A_f \frac{\epsilon_f}{\epsilon_y}) f_y$$

Introducing equivalent steel  $A_{eq} = (A_s + A_f \frac{\epsilon_f}{\epsilon_y})$ ,  $\Sigma T = A_{eq} f_y$  acting at location of  $d_{eq}$

$$M_n = \Sigma T (d_{eq} - a_{beq}/2) = A_{eq} f_y (d_{eq} - a_{beq}/2)$$

From equilibrium

$$C = T, \quad 0.85 f_c' b a_{beq} = A_{eq} f_y, \quad a_{beq} = \frac{A_{eq} f_y}{0.85 f_c' b}$$

$$\text{And } \rho_{eq} = \frac{A_{eq}}{b d_{eq}} = 0.85 \beta_1 \left( \frac{\epsilon_{cu}}{\epsilon_{eq} + \epsilon_{cu}} \right) f_c' / f_y$$

By limiting the carbon fiber strain to about 70% of its ultimate as shown in Fig 2 which is about 0.01

$$\text{Taking } \epsilon_y = 0.01 \text{ and } \epsilon_{cu} = 0.003, \quad c_{beq} = \left( \frac{\epsilon_{cu}}{\epsilon_{eq} + \epsilon_{cu}} \right) d_{eq} = 0.231 d_{eq}, \quad a_{beq} = 0.85 c_{beq} \cong 0.2 d_{eq}$$

$$\text{Introducing } R, \quad M = R b d_{eq}^2, \quad M = 0.9 f_y A_{eq} d_{eq}, \quad R = \frac{0.9 f_y A_{eq}}{b d_{eq}}, \quad \rho_{eq} = \frac{A_{eq}}{b d_{eq}}, \quad R = 0.9 \rho_{eq} f_y$$

Since  $\epsilon_f = \frac{f_{fu}}{\epsilon_{fu}}$  and  $\epsilon_y = \frac{f_y}{\epsilon_y}$ , Applying the limiting strain of carbon fiber to ( $\epsilon_f = 0.70 \epsilon_{fu}$ ) and equal

$$\text{to } \epsilon_y \text{ then term } \frac{\epsilon_f}{\epsilon_y} \text{ becomes } (0.70 \frac{f_{fu}}{f_y}), \text{ And } \rho_{eq} \text{ (in terms of stress)} = \frac{(A_s + 0.70 A_f \frac{f_{fu}}{f_y})}{b d_{eq}}$$

## APPENDIX B

### Application of Moment Capacity Chart in Design

In strengthening of any rectangular section,  $A_s, f_y, b, d, f_{fu}$  are known

- 1- Find required ultimate moment for the strengthened beam.
- 2- Assume an approximate equivalent depth of reinforcement.
- 3- Find  $R = \frac{M_u}{b d_{eq}^2}$
- 4- From the chart find  $\rho_{eq}$
- 5- Since  $A_s$  is known, find  $A_f$
- 6- Calculate  $a = \frac{(A_s + A_f \frac{\epsilon_f}{\epsilon_y}) f_y}{0.85 f_c' b}$
- 7- Find  $M_u = (A_s + 0.70 A_f \frac{f_{fu}}{f_y}) (d_{eq} - \frac{a}{2})$
- 8- If  $\phi M_u \geq M_{required} \rightarrow \text{ok} \quad \phi = 0.9$
- 9- If  $\phi M_u \leq M_{required} \rightarrow$  then new trial should be considered
- 10- Find new  $a_{beq} = \beta_1 c_{beq} = \beta_1 \left( \frac{\epsilon_{cu}}{\epsilon_{eq} + \epsilon_{cu}} \right) d_{eq}$  with  $\epsilon_{cu} = 0.003$  and  $\epsilon_{eq}$  Increased

From 0.01 to 0.011 and find new  $\phi M$ , In most cases the first or second trial should be sufficient.

## **APPENDIX C**

Table 1 Database of Abdullah [16]

JZS (2016) 18 - 2 (Part-A)

Reference *	NO.	Beam notation	Width 'b' (mm)	d <sub>f</sub> (mm)	d <sub>s</sub> (mm)	f <sub>c</sub> ' (Mpa)	f <sub>tu</sub> (Mpa)	f <sub>sy</sub> (Mpa)	E <sub>f</sub> (Mpa)	E <sub>s</sub> (Mpa)	A <sub>f</sub> (mm <sup>2</sup> )	A <sub>s1</sub> (mm <sup>2</sup> )	Observed modes of failure	Types of FRP	M <sub>exp</sub> (KN.mm)	
Hawileh 2012	1	SC12 (VC30)	150	268	238	35.1	1875	600	146000	210000	113.1	226.1	SY-CC	CFRP-bars	65400	
Kotynia 2012	2	NIISB/40/80	150	392	354	41.58	1893	542	169000	209000	36	401.9	SY,RS	CFRP-strip	78000	
yost et al. 2007	3	6-1Fa	152.4	176	144	37.2	1648	490	136000	200000	37.5	401.9	CC	CFRP-strip	30267.77	
	4	6-1Fb	152.4	176	144	37.2	1648	490	136000	200000	37.5	401.9	CC	CFRP-strip	28329.56	
	5	6-2Fa	152.4	176	144	37.2	1648	490	136000	200000	75	401.9	CC	CFRP-strip	30462.81	
	6	6-2Fb	152.4	176	144	37.2	1648	490	136000	200000	75	401.9	CC	CFRP-strip	32839.86	
	7	9-1Fa	229	176	146	37.2	1648	510	136000	200000	37.5	398	CC	CFRP-strip	34400.18	
	8	9-1Fb	229	176	146	37.2	1648	510	136000	200000	37.5	398	CC	CFRP-strip	34046.67	
	9	9-2Fa	229	176	146	37.2	1648	510	136000	200000	75	398	CC	CFRP-strip	45163.95	
	10	9-2Fb	229	176	146	37.2	1648	510	136000	200000	75	398	CC	CFRP-strip	43664.58	
	11	12-1Fa	305	176	146	37.2	1648	510	136000	200000	37.5	398	RF	CFRP-strip	36070.21	
	12	12-1Fb	305	176	146	37.2	1648	510	136000	200000	37.5	398	RF	CFRP-strip	37801.19	
	13	12-2Fa	305	176	146	37.2	1648	510	136000	200000	75	398	CC	CFRP-strip	41202.2	
	14	12-2Fb	305	176	146	37.2	1648	510	136000	200000	75	398	CC	CFRP-strip	50917.63	
	Teng et al. 2006	15	B2900	150	289	256	35.2	2068	532	131000	210000	32	226.1	CC	CFRP-strip	59880
	Jung et al. 2005	16	NSM-PL-15	200	293	255	31.3	2453	426	165490	200000	21	214	RF	CFRP-strip	41207.25
17		ROD-MI-20	200	288	255	31.3	1878	426	121420	200000	63.6	214	RF	CFRP-rod	55755	
18		PL-MI-20	200	288	255	31.3	2453	426	165490	200000	35	214	RF	CFRP-strip	51828	
El-Hacha and Rizkalla 2004	19	B2	300	291	237	45	1525	400	140000	200000	64	650.1	RF	CFRP-strip	62062	
	20	B3	300	288	237	45	2000	400	150000	200000	60	650.1	RF	CFRP-strip	68875	
Hassan and Rizkalla 2003	21	B5	150	285	235	48	2000	400	150000	200000	30	510.3	RF	CFRP-strip	49375	
	22	B6	150	285	235	48	2000	400	150000	200000	30	510.3	RF	CFRP-strip	46875	
	23	B7	150	285	235	48	2000	400	150000	200000	30	510.3	RF	CFRP-strip	50000	
	24	B8	150	285	235	48	2000	400	150000	200000	30	510.3	RF	CFRP-strip	50000	

Observed modes of failure: CC: Concrete Crushing, RF: Rupture of FRP, SY: Steel Yielding, RS: Rupture of Steel bar [16].

