Journal homepage www. jzs.uos.edu.krd
Journal of Zankoi Sulaimani
Part-A- (Pure and Applied Sciences)

S_g- Open Sets in Topological Spaces

Halgwrd M. Darwesh[1] & Nawroz O. Hessean[2]

1 College of Commerce Dept. of Computer & Statistics. University of Sulaimani, Sulaimani, Kurdistan Region, Iraq.
2 Department of Information and Technology, Kurdistan Institution for Strategic students and scientific Research, Sulaimani, Kurdistan Region, Iraq
e-mail: halgwrd.darwesh@univsul.edu.iq

Article info

Original: 05 Nov. 2014
Revised: 04 Dec. 2014
Accepted: 10 Dec. 2014
Published online:
20 March 2015

Key Words:
Semi-open sets
g-closed sets
S_g-open sets.

Abstract

In this paper, we introduce a new type of semi-open sets which we call s_g-open sets, and then we study some its basic properties and characterizations, further, we investigate some relationships with some other types of sets.

Introduction and Preliminaries

The notion of semi-open sets and generalized closed (briefly, g-closed)sets introduced by Levine in 1963 [19] and 1970 [20] respectively. Mathematicians gave in several papers different and interesting new types of sets such as; Veličoko [30] in 1968, introduced δ- open and θ-open sets to investigate some characterizations of H-closed spaces, Joseph, Kwach [14] and DiMaio , Noiri [10] defined θ-semi – open and semi-θ-open sets to investigate some properties of S-closed and s-closed spaces, respectively. In 1997 Park and et. al. [24] used the notion δ- open sets to define δ- semiopen sets, Darwesh [8] used δ- semiopen and closed sets to define the notion of δs_c-open sets, However, Darwesh and Shareef [9] defined p_g-open sets by using the notions of pre-open sets and g-closed sets. Kalaf and et. al. [17] (resp., [26], [16] and [18]) used the notion of semi-open sets and closed (resp. pre-closed, semi-closed and β-closed) sets to define and introduce new types of semi-open sets namely s_c-open (resp. s_p-open, s_s-open and s_β-open) sets. However, in the present paper, we introduce a new class of sets called s_g-open by using semi-open and g-closed sets of Levine, which is strictly placed between the classes of s_c-open sets and semi-open sets. Some of its basic properties and relationships with some other types of sets are given. Throughout the present paper (X, τ) (simply, X) denotes a topological space on which no separation axioms are assumed unless explicitly stated. For any subset A of a space X , the ClA , $sClA$, $IntA$ and $Int_\delta A$ denote the closure, semi-closure, interior and δ-interior of A, respectively.

Definition 1.1 A subset A of a space X is said to be semi-open [19] (resp., pre-open [22], α-open[23], β-open [1], δ-semiopen[24] and regular open [29]) set if $A \subseteq ClIntA$ (resp. $A \subseteq IntClA$, $A \subseteq IntClIntA$, $A \subseteq ClIntClA$, $A \subseteq ClInt_\delta A$ and $A = IntClA$). The complement of a semi-open (resp. pre-open, α-open, β-open, δ- semiopen and regular open) set is said to be semi-closed (resp., pre-closed, α-closed, β-closed, δ-semiclosed and regular closed).

Definition 1.2 A subset A of a space X is said to be δ-open [30] (resp. θ-open [30], θ-semi-open[15] and semi-θ-open [10]) if for each $x \in A$, there exists an open(resp., open, semi-open and semi-open) set G such that $x \in G \subseteq IntClG \subseteq A$ (resp. $x \in G \subseteq ClG \subseteq A$, $x \in G \subseteq ClG \subseteq A$ and $x \in G \subseteq sClG \subseteq A$).The complement of each θ-semi-open (resp. semi-θ-open) set in X is θ-semi-closed (resp., semi-θ-closed) set in X.

Definition 1.3[20] A subset A of a space X is said to be g-closed if $ClA \subseteq U$, whenever U is open and $A \subseteq U$.

Definition 1.4 A semi-open subset A of a space X is said to be s_c-open[17] (resp. s_p-open [26], s_s-open [16] and s_β-open [18])if for each $x \in A$ there exists a closed (resp., pre-closed, semi-closed and β-closed) set F such that $x \in F \subseteq A$.

Definition 1.5 [8] A δ-semiopen subset A of a space X is said to be δs_c-open if for each $x \in A$ there exists a closed set F such that $x \in F \subseteq A$.

Definition1.6 [4] A pre-open subset A of a space X is called p_s-open (resp. p_g-open [9]) if for each $x \in A$, there exists a semi-closed (resp., g-closed) set F such that $x \in F \subseteq A$.

The family of all semi-open (resp., pre-open, β-open , δ-semiopen, regular open, regular-closed, g-closed, δ-open , θ-open,θ-semi-open, semi-θ-open, s_c-open, s_p-open, s_s-open, s_β-open, δs_c-open, p_s-open and p_g-open) subsets of X are denote by $SO(X)$ (resp $PO(X)$, $\beta O(X)$, $\delta SO(X)$,$RO(X)$, $RC(X)$, $GC(X)$, $\delta O(X)$, $\theta O(X)$, $\theta SO(X)$, $S\theta O(X)$, $S_C O(X)$, $S_P O(X)$, $S_S O(X)$, $S_\beta O(X)$,$\delta S_C O(X)$, $P_s O(X)$ and $P_g O(X)$)

Theorem 1.7 [7] Let(X, τ)be any space. If $A \in \tau$ *and* $B \in SO(X)$, then $A \cap B \in SO(X)$.

Theorem 1.8 [20] Let$A$ be a g-closed set and suppose that F is a closed set. Then $A \cap F$ is a g-closed set.

Theorem 1.9 [20] If A and B are g-closed, then $A \cup B$ is g-closed.

Theorem 1.10 [20] Let $A \subseteq Y \subseteq X$ and suppose that A is g-closed in X. Then A is g-closed in Y.

Theorem 1.11 Let A be a subset of a subspace$Y$ of a space X.

1- If $A \in SO(X)$, then $A \in SO(Y)$. [19]
2- If $A \in SO(Y)$and $Y \in SO(X)$, then $A \in SO(X)$. [3]

Theorem 1.12 [20] Suppose that $B \subseteq A \subseteq X$, B is a g-closed set relative to A and that A is a g-closed subset of X, then B is g-closed in X.

Theorem 1.13 [19] Let A be semi-open in a space X and suppose that $A \subseteq B \subseteq ClA$. Then B is semi-open.

Theorem 1.14 [19] Let A be any subset of a space X. Then $A \in SO(X)$) if and only if $Cl(A) = ClInt(A)$.

Theorem 1.15 Let X_1 and X_2 be topological spaces and $X = X_1 \times X_2$ be the topological product.

1- If $A_1 \in SO(X_1)$and $A_2 \in SO(X_2)$, then $A_1 \times A_2 \in SO(X)$. [19]
2- If $A_1 \in GC(X_1)$and $A_2 \in GC(X_2)$, then $A_1 \times A_2 \in GC(X)$. [5]

Definition 1.16 [3] A space (X, τ) is said to be $s^{**}-$normal if for every semi-closed set F and every semi-open set G containing F, there exists an open set H such that $F \subseteq H \subseteq ClH \subseteq G$.

Lemma 1.17 [33] If X is s^{**} - normal, then $S\theta O(X) = \theta O(X) = \theta SO(X)$.

Definition 1.18 [32] A space X is said to be extermally disconnected if the closure of every open set in X is open.

Theorem1.19 [23] A space X is extermally disconnected if and only if $SO(X)$ is a topology on X.

Theorem 1.20 The following conditions are equivalent for the extremally disconnected space X:

1-$RO(X) = RC(X)$ [4]
2- $\delta O(X) = \theta SO(X)$ [33]

Lemma 1.21 [31] A space X is extremally disconnected if and only if $Cl(A) \cap Cl(B) = Cl(A \cap B)$ for all open subsets A and $B$of X.

It is easy to see that

Corollary 1.22 If X is an extremally disconnected space, then $Cl(A) \cap Cl(B) = Cl(A \cap B)$for any semi-open subsets$A$ and $B$of X.

Lemma 1.23 [12] For a subset A of a space (X, τ), the following conditions are equivalent:

1- $A \in RO(X)$.
2- $A \in \tau \cap SC(X)$.
3- $A \in \alpha O(X) \cap SC(X)$.
4- $A \in PO(X) \cap SC(X)$.

Definition 1.24 [21] A subfamily τ^* of X is said to be a supra topology on X if:
1- $X, \emptyset \in \tau^*$.
2- If $A_i \in \tau^*$ for all $i \in J$, then $\cup A_i \in \tau^*$.

Theorem 1.25 [25]Let$X$ be a space. Then the following statements are equivalent:
1- $X$is a T_{gs}-space
2- Every g-closed subset of X is β-closed
3- Every g-closed subset of X is preclosed.

Theorem1.26 [25] A space X is nodeg and extermally disconnected if and only if every semi-closed subset of X is g-closed.

Definition1.27 [11] A space X is called strongly irresolvable if and only if no non-empty open set is resolvable.

Theorem1.28 [11] For a space X the following conditions are equivalent:
1- X is strongly irresolvable.
2- Every pre-open subset is semi-open.
3- Every pre-open subset is α-open.

Theorem1.29 [11] A topological space X is submaximal if and only if every preopen set is open.

Definition 1.30 [2] A space X is said to be locally indiscrete if every open subset of X is closed.

Definition 1.31 [20] A space X is said to be $T_{\frac{1}{2}}$ - space if and only if every g-closed set is closed.

Definition 1.32 [6] A space X is said to be an $ST_{\frac{1}{2}}$ - space if every semi-closed set of X is closed in X.

S_g- open sets

Definition 2.1A semi-open set A of a space X is said to be an S_g-open set if for each $x \in A$, there exists a g-closed set F such that $x \in F \subseteq A$. The family of S_g–open subsets of X is denoted by $S_gO(X)$.

Proposition 2.2 A subset A of a space X is S_g-open if and only if A is semi-open and A is a union of g-closed sets.

Proof. Obvious.

Proposition 2.3 If every singleton set is g-closed in a space X, then $SO(X) = S_gO(X)$.

Proof. Obvious.

It is clear from the Definition 2.1 that every S_g-open subset of a space X is semi-open set, but the converse is not true in general as it is shown in the following example:

Example 2.4 Let $X = \{a,b,c\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}\}$.Then
$SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$ and $GC(X) = \{\emptyset, X, \{c\}, \{a,c\}, \{b,c\}\}$ and $S_gO(X) = \{\emptyset, X, \{a,c\}, \{b,c\}\}$. The subset $\{a,b\}$ is semi-open but not S_g-open.

The following result shows that any union of S_g-open subsets in a space X is also S_g–open.

Proposition 2.5 Let $\{A_\alpha : \alpha \in \Delta\}$ be a collection of S_g-open sets in a space X. Then $\cup\{A_\alpha : \alpha \in \Delta\}$ is S_g-open.

Proof. Let $A_\alpha$be an S_g-open set for each $\alpha \in \Delta$. Then A_α is semi-open for each α, and then $\cup\{A_\alpha : \alpha \in \Delta\}$ is semi-open. Suppose that $x \in \cup\{A_\alpha : \alpha \in \Delta\}$, this implies that there exists $\alpha_0 \in \Delta$, such that $x \in A_{\alpha_0}$ and since A_{α_0} is an S_g-open sets, so there exists a g-closed set F such that $x \in F \subseteq A_{\alpha_0} \subseteq \cup\{A_\alpha : \alpha \in \Delta\}$ Therefore, $\cup\{A_\alpha : \alpha \in \Delta\}$ is an S_g-open set.

In Example 2.4, we have $\{a,c\}$ and $\{b,c\} \in S_gO(X)$ but $\{a,c\} \cap \{b,c\} = \{c\} \notin S_gO(X)$ .This means that the intersection of two S_g-open sets may not be S_g-open in general .

Proposition 2.6 The set A is S_g-open in a space X if and only if for each $x \in A$, there exists an S_g-open set B such that $x \in B \subseteq A$.

Proof. Let A be S_g-open in X. Then for each $x \in A$, put $B = A$ is an S_g-open set containing x such that $x \in B \subseteq A$.
Conversely, suppose that for each $x \in A$ there exists an S_g-open set B_x such that $x \in B_x \subseteq A$, then $A = \bigcup_{x\in A}\{x\} \subseteq \bigcup_{x\in A}\{B_x\} \subseteq A$.Thus $A = \bigcup_{x\in A}\{B_x\}$, where $B_x \in S_gO(X)$ for each x . Then A is a union of S_g-open sets. Hence by Proposition 2.5, A is S_g-open.
From Proposition 2.6 and Example 2.4 we notice that the family $S_gO(X)$ of a space X form a supra topology on X and need not be a topology on X in general.
Theorem 2.7 If the family of all semi-open sets of the space X is a topology on X and the intersection of any two g-closed sets of X is g-closed,then the family of all S_g-open subsets of X forms a topology on X.
Proof. Clearly $\emptyset$ and $X \in S_gO(X)$ and by Proposition 2.5, the union of any subfamily of $S_gO(X)$ is also in $S_gO(X)$. So we have to show the intersection of two S_g-open sets is S_g-open. Let A and B be two S_g-open sets. Since $SO(X)$ is a topology on X, then $A \cap B \in SO(X)$. Let $x \in A \cap B$. Then $x \in A$ and $x \in B$. Since $A, B \in S_gO(X)$, then there exist g-closed sets F and E such that $x \in F \subseteq A$ and $x \in E \subseteq B$. Since $F \cap E$is g-closed and $x \in F \cap E \subseteq A \cap B$. Then $A \cap B \in S_gO(X)$. Thus $S_gO(X)$ is a topology on X.
Corollary 2.8 Let X be an extremely disconnected spacesuch that the intersection of two g-closed sets is also g-closed. Then $S_gO(X)$ forms a topology on X.
Proof. Follows from Theorem 2.7 and Theorem 1.19
The following example shows that there exists a family of S_g-open sets which forms a non-trivial topology on X.
Example 2.9 Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{X, \{a\}, \{c\}, \{b, d\}, \{a, c\}, \{a, b, d\}, \{c, b, d\}\}$. Then $SO(X) = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$, $GC(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, d\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, \{a, c, d\}\}$ and $S_gO(X) = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$. So $S_gO(X) = SO(X)$and it is a non-trivial topology on X. Also we see that, if X is extermally disconnected or equivalently $SO(X)$ is a topology on X, then $S_gO(X)$ may not be a topology on X as it shown in the following example
Example 2.10 Let $X = \{a, b, c\}$ and $\tau = \{\emptyset, X, \{a\}\}$. Then $SO(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$ is a topology on X but $S_gO(X) = \{\emptyset, X, \{a, b\}, \{a, c\}\}$ is not a topology on X.
The relation of S_g-open sets to some other types of sets is illustrated in the following:
Proposition 2.11 Every θ-open (resp. θ −semi open) set of X is an S_g-open set .
Proof. Let A be an arbitrary θ–open (resp. θ-semi-open) set in X. Then A is a semi-open set and for each $a \in A$, there exists an open (resp. semi-open) set G such that $a \in G \subseteq ClG \subseteq A$. Since ClG is closed then it is g-closed, so A is S_g-open.
The following example shows that the converse of Proposition 2.11 is not true in general.
Example 2.12 Let $X = \{a, b, c\}$and $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$. Then $SO(X) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$, $GC(X) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$, $S_gO(X) = \{\emptyset, \{a, c\}, X\}$, $\tau_\theta = \{\emptyset, X\}$ and $\theta SO(X) = \{\emptyset, X\}$. So we have $\{a, c\}$ is S_g -open but it is neither θ-open nor θ-semi-open.
Corollary 2.13
1- Every regular closed set of a space X is S_g-open.
2- Every δS_c-open andS_c–open of a space X is S_g-open.
Proof. Obvious.
The converse of neither parts of Corollary 2.13 is true, in general. Since in Example 2.10, we have $S_gO(X) = \{\emptyset, \{a, c\}, \{a, b\}, X\}$and $S_cO(X) = \{\phi, X\} = \delta S_cO(X)$, so $\{a, b\} \in S_gO(X)$ but $\{a, b\} \notin S_cO(X)$. However, in Example 2.12, we have $\{a, c\} \in S_gO(X)$ but$\{a, b\} \notin RC(X)$.
Example 2.14 $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ and $SO(X) = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\{, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$,$S_gO(X) = \emptyset, X, \{b, d\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ and $S_pO(X) = \{\emptyset, X, \{a, c, d\}, \{b, c, d\}, \{b, c\}, \{a, d\}\}$. Thus $\{a, b, d\} \in S_gO(X)$ but $\{a, b, d\} \notin S_pO(X)$. However, in Example 2.1.13 [26], we have $PC(X) = P(X) - \{\{b, c\}, \{a, b, c\}\}$ and $S_pO(X) = \{\emptyset, X, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$, However, $S_g(X) = \{\phi, X\}$, then $\{b, c\} \in$

$S_pO(X)$ but $\{b,c\} \notin S_gO(X)$. Hence the concepts of S_g-openness and S_p-openness are independent topological concepts.
Also, the following examples show that the concepts of S_g-openness and p_g-openness are independent topological concepts.

Example 2.15 Let $X = \{a, b, c\}$ and $\tau = \{\phi, X\}$. Then $GC(X) = P(X) = PO(X) = P_gO(X)$, but $S_CO(X) = \{\phi, X\} = SO(X) = S_gO(X)$. so the set $\{a\}$is p_g-open but not s_g-open.

Example 2.16 Consider the usual topological space $(R, \Im_U)$[28]. The subset $A = [0,1)$ is s_g-open but not p_g-open.

Proposition 2.17 If a topological space X is locally indiscrete, then $SO(X) = S_gO(X)$.

Proof. Let A be a semi-open set in X, then $A \subseteq ClIntA$. Since X is locally indiscrete, then $IntA$ is closed and hence $IntA = ClIntA$, which implies that A is regular closed. Therefore, by Corollary 2.13, $A \in S_gO(X)$.

Corollary 2.18 If a topological space $(X.\tau)$ is locally indiscrete, then $S_gO(X) = \tau$.

Proof.Follows from Proposition2.17.

Corollary 2.19 If the space X is locally indiscrete, then $IntA$ is an S_g-open subset of X, for any subset A of X.

Proposition 2.20 If a space X is aT_1-space, then $SO(X) = S_cO(X) = S_gO(X)$.

Proof. Let A be a semi-open set in X. If $A = \emptyset$,then $A \in S_gO(X)$. If $A \neq \emptyset$, then for each $x \in A$. Since the space X is a T_1-space, then every singleton set is closed and hence $x \in \{x\} \subseteq A$. Therefore, $A \in S_cO(X)$.Hence $SO(X) \subseteq S_cO(X) \subseteq S_gO(X)$, but since$S_gO(X) \subseteq SO(X)$ in general. Therefore, $SO(X) = S_cO(X) = S_gO(X)$.

Remark 2.21

1- Since every open set is semi-open, it follows that, if a topological space (X, τ) is a T_1 -space, then $\tau \subseteq S_gO(X)$.
2- In a space X if $A \in SO(X)$, then $ClA \in S_gO(X)$.

For the definitions of completely regular and regular spaces, see [27]

Proposition 2.22 If $(X, \tau)$is a completely regular or regular space, then $\tau \subseteq S_gO(X)$.

Proof. Since every completely regular space is regular, so we have only to show the case that the space (X, τ) is regular. For this, let A be any open subset of a regular space X. Then A is semi-open.If $A = \emptyset$, then $A \in S_gO(X)$. If $A \neq \emptyset$, since X is regular, then for each $x \in A \subseteq X$, there exists an open set G such that $x \in G \subseteq ClG \subseteq A$.Since ClG is g-closed, then$A \in S_gO(X)$, therefore $\tau \subseteq S_gO(X)$.

Proposition 2.23 If a space X is a $T_{\frac{1}{2}}$ - space, then $S_gO(X) = S_cO(X)$.

Proof. By part 2 of Corollary 2.13, $S_cO(X) \subseteq S_gO(X)$, Since a space X is a$T_{\frac{1}{2}}$ - space , then every g-closed set is closed. Hence $S_gO(X) \subseteq S_cO(X)$. Therefore, $S_gO(X) = S_cO(X)$.

Corollary 2.24 If a space X is a $T_{\frac{1}{2}}$ - space, then:

1- $S_gO(X) \subseteq S_sO(X)$
2- $S_gO(X) \subseteq S_\beta O(X)$
3- $S_gO(X) \subseteq S_pO(X)$.

Proof. Since in a $T_{\frac{1}{2}}$-space, every g-closed set is closed and every closed is semi-closed, β-closed and pre-closed.

Proposition 2.25 Let X be a space and $A, B \subseteq X$. If $A \in S_gO(X)$ and B is clopen, then $A \cap B \in S_gO(X)$.

Proof. Let $A \in S_gO(X)$ and B is clopen. Then A is a semi-open set, this by Theorem1.7 implies that $\cap B \in SO(X)$. Now, if $x \in A \cap B$ then $x \in A$and $x \in B$. ,Therefore, there exists a g-closed set F such that $x \in F \subseteq A$. Since B is clopen, so B is closed and then by Theorem 1.8, $F \cap B$ is g-closed, such that $x \in F \cap B \subseteq A \cap B$. Thus $A \cap B \in S_gO(X)$.

Proposition 2.26 Let X be an extermally disconnected topological space. If $A \in S_gO(X)$and $B \in RC(X)$, then $A \cap B \in S_gO(X)$.

Proof. Let $A \in S_gO(X)$and $B \in RC(X)$, then A is semi-open and since X is extermally disconnected, then by part 1 of Theorem 1.20,$B \in RO(X)$, this implies that B is clopen, so by Proposition 2.25, $A \cap B \in S_gO(X)$.

Proposition 2.27 Let X be an extermally disconnected space. If $A \in \delta O(X)$, then $A \in S_gO(X)$.

Proof. Let $A \in \delta O(X)$. Since the space X is extermally disconnected ,then by part 2 of Theorem1.20, $\delta O(X) = \theta SO(X)$. Hence $A \in \theta SO(X)$. But by Proposition 2.11, we have $\theta SO(X) \subseteq S_gO(X)$. Therefore, $A \in S_gO(X)$.

Corollary 2.28 Let X be an extermally disconnected space. If $A \in RO(X)$, then $A \in S_gO(X)$.

Proof. Follows directly from Proposition2.27 and the fact that $RO(X) \subseteq \delta O(X)$.

Proposition 2.29 Let X_1 and X_2 be topological spaces and $X = X_1 \times X_2$ be the topological product. If $A_1 \in S_gO(X_1)$ and $A_2 \in S_gO(X_2)$.Then $A_1 \times A_2 \in S_gO(X)$.

Proof. Let $A_1 \in S_gO(X_1)$ and $A_2 \in S_gO(X_2)$.Then $A_1 \in SO(X_1)$ and $A_2 \in SO(X_2)$ and so by Theorem1.15,$A_1 \times A_2 \in SO(X)$. If A_1 or $A_2$is empty, then $A_1 \times A_2 = \emptyset \in S_gO(X)$. If A_1 and A_2 are not empty, then for each $(x, y) \in A_1 \times A_2$, we have $x \in A_1$ and $y \in A_2$. then there exists g-closed sets F and E in X_1 and X_2 respectively, such that $x \in F \subseteq A_1$ and $y \in E \subseteq A_2$. Therefore, $(x, y) \in F \times E \subseteq A_1 \times A_2$. Since F is g-closed in $X_1$and E is g-closed in X_2, then by Theorem1.15, $F \times E$ is g-closed in X, and hence $A_1 \times A_2 \in S_gO(X)$.

Proposition 2.30 Let (X, τ) be an S^{**}-normal space .If $A \in S\theta O(X)$, then $A \in S_gO(X)$.

Proof.Let $A \in S\theta O(X)$. Since a space X is S^{**}--normal, then by Lemma 1.17, $S\theta O(X) = \theta SO(X)$. Hence $A \in \theta SO(X)$. But $\theta SO(X) \subseteq S_gO(X)$ by Proposition 2.11.Thus $A \in S_gO(X)$.

Proposition 2.31 Let A be a subset of a subspace Y of a space X. If $A \in S_gO(X)$, then $A \in S_gO(Y)$.

Proof. Let $A \in S_gO(X)$. Then $A \in SO(X)$ and since $A \subseteq Y$ then by Theorem1.11, $A \in SO(Y)$. So, for each $x \in A$, there exists a g-closed set F in X such that $x \in F \subseteq A$. Since $F \subseteq Y$, then by Theorem1.10, F is g-closed in Y. Hence $A \in S_gO(Y)$.

In the Example 2.4, if we put $Y = \{a, b\}$, then $\tau_Y = \{\emptyset, Y, \{a\}, \{b\}\} = P(Y)$ is a relative topology on Y. So $SO(Y) = \tau_Y = P(Y)$ and $S_gO(Y) = \tau_Y = P(Y)$. Hence $\{a\} \in S_gO(Y)$but $\{a\} \notin S_gO(X)$. This means that the converse of Proposition 2.31 is not true, in general.

In the following example, we show that If A is an S_g-open set in X and Y is a subspace of X, then $A \cap Y$ is not an S_g-open set in Y.

Example 2.32 Let $X = \{a, b, c, d\}$and $\tau = \{\emptyset, X, \{a\}, \{c\}, \{a, b, c\}, \{a, c\}\}$. Then $S_gO(X) = \{\emptyset, X, \{a, d\}, \{c,d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$. If we put $Y = \{a, b, c\}$ then $\tau_Y = \{\emptyset, Y, \{a\}, \{c\}, \{a, b\}, \{a, c\}\}$ is relative topology on Y, and then $S_gO(Y) = \{\emptyset, Y, \{c\}, \{a, b\}, \{a, c\}\}$ but $\{a, d\} \cap Y = \{a\}$ is not an S_g-open set in Y.

Proposition 2.33 Let A be a subset of a subspace Y of a space X. If $A \in S_gO(Y)$and $Y \in RC(X)$, then $A \in S_gO(X)$.

Proof. Let $A \in S_gO(Y)$. Then $A \in SO(Y)$ and for each $x \in A$, there exists a g-closed set F in Y such that $x \in F \subseteq A$. Since $Y \in RC(X)$, then $Y \in SO(X)$ and Y is a closed set in X. Then by Theorem 1.11, $A \in SO(X)$ and by Theorem1.12, F is g-closed in X. Hence $A \in S_gO(X)$.

From Proposition 2.31 and Proposition 2.33 we obtain the following results:

Corollary 2.34 If A is a subset of a subspace Y of a space X such that $Y \in RC(X)$, then $A \in S_gO(Y)$ if and only if $A \in S_gO(X)$.

Corollary 2.35 If A is a subset of a clopen subspace Y of a space X, then $A \in S_gO(Y)$ if and only if $A \in S_gO(X)$.

Corollary 2.36 Let Y is a clopen subspace Y of a space X. If $A \in S_gO(X)$, then $A \cap Y \in S_gO(Y)$.

Corollary 2.37 If a space X is extermally disconnected for all $A$and $B \in S_gO(X)$, then $Cl(A \cap B) = ClA \cap ClB$.

Proof. Let $A$and $B \in S_gO(X)$. Then A and $B \in SO(X)$.Therefore, by Corollary 1.22 $Cl(A \cap B) = ClA \cap ClB$.

Theorem 2.38 Let Abe anS_g-open set in a space X. If $A \subseteq B \subseteq ClA$and $B \in PC(X)$, then B is anS_g-open set in X.

Proof. Since A is S_g-open then A is semi-open by Theorem1.13, B is semi-open set, and since B is pre-closed, which implies that B is regular closed. Then by Corollary 2.13, B is anS_g-open set.

Proposition 2.39 For any subset A of a space X, the following conditions are equivalent:

1- $A$is regular closed.
2- $A$is closed and S_g-open.

3- $A$is closed and semi-open.
4- $A$is closed and β-open.
5- Aisα-closed and β-open.
6- $A$is pre-closed and β-open.

Proof. Follows from Lemma1.23, and the fact that every semi-open set is β-open.

Recall that if a space X is extremally disconnected, then every semi-open is pre-open[15].

Proposition 2.40 If the space X is extremally disconnected, then every S_g-open subsetof X is p_g- open and hence pre-open in X.

Proof. Let A be an s_g-open set, then A is semi-open and it is a union of g-closed sets. Since every semi-open in an extremally disconnected space is pre-open, then A is pre-open and it is a union of g-closed sets, therefore, A is a p_g- open set.

Proposition 2.41 If X is a T_{gs}-space, then every s_g-open set is s_p-open and s_β-open

Proof. Follows from Definition 1.4 and Theorem 1.25.

Proposition 2.42 If X is a nodeg and extremally disconnected or (an $ST_{\frac{1}{2}}$- space) , then every s_s-open set is an s_g-open set and every p_s-open set is p_g-open.

Proof. Follows from Definition 1.4 and Theorem 1.26 (or Definition 1.32).

Proposition 2.43 If X is a strongly irresolvable space, then every p_g-openset iss_g-open.

Proof. Follows from Definition 1.6 and Theorem 1.28.

Proposition 2.44

If a space X is submaximal, then every s_p-open is s_g-open.

Proof. Follows from Definition 1.4 and Theorem1.29.

References

[1] Abd-El-Monsef M. E. El-Deeb S. N. and Mahmoud R. A. (1983), β-open sets and β-continuous mappings, Bull. Fac. Assint, Vol. 12, 77-90.

[2] Abdulla M., Topological property of θ-semi-open sets, Internat. J. of Pure & Appl. Sci. and Tech.

[3] Ahmed N. K. (1990), On some Types of Separation Axioms, M. Bc. Thesis, Salahadin Univ., Arbil,.

[4] Asaad B. A. (2007), Utilization of Some Types of Per-open Sets in Topological Spaces, M.Sc. Thesis, Univ. of Dohuk.

[5] Balasubramanian S. (2012), Minimal g-open sets and Maximal g-closed sets, Asian J. of current engineering and Maths, 69-73.

[6] Benchalli S. S. (2011), Semi-totally Continuous functions in Topological Spaces, Internat. Math. Forum, Vol. 6, No. 10, 479-492.

[7] Crossely S. G. and Hildebrand S. K. (1971), Semi-closure, Texas J. Sci., Vol.22, 99-112.

[8] Darwesh H. M. (2011), A New Type of Semi-Open Sets and Semi-Continuity in Topological Spaces, Zanco J. of Pure and Appl. Sci., Vol. 23, No. 2, 82-92.

[9] Darwesh, H. M., and Sareef, H. A. (2014) ,p_g-open Sets in Topological Spaces, submitted.

[10] DiMaio G. and Noiri T. (1987) , On s-closed spaces, Indian J. Pure Appl. Math., Vol. 18, No. 3, , 226-233.

[11] Dontchev J. (1998), Survey on preopen sets, ArXvi Math., Vol. 1, , 1-18.

[12] Dontchev J. and Noiri T. (1999), Contra-semi continuous functions, Math. Pannon, Vol. 10, No. 2, , 159-168.

[13] El-Deeb S. N., Hasanein I. A., Mashhour A. S. and Noiri T. (1983), On P-regular Spaces, Bulletin Mathematique de la societe des Sciences Mathematiques de Roumanie, Vol. 27, No. 4, 311-315.

[14] Guo T. Y. (1981), A characterization of extermally disconnected spaces, J. central china Normal Univ. Nature. Sci., Vol. 12, No. 2, 169-170.

[15] Joseph J. E. and Kwack M. H. (1980), On S-closed spaces, Proc. Amer. Math. Soc.,Vol. 80, No. 2, 341-348.

[16] Khalaf A. B. Majeed A. H. and Jami J. M. (2014), Ss-open sets and Ss -continuous Functions, Int. J. of Advanced Math. Sci., Vol. 2, No. 1, 71-80.
[17] Khalaf A. B. and Ameen Z. A. (2010), S_c-open sets and S_c-continuity in topological spaces, J. of Advanced Research in Pure Math. Vol. 2, No. 3, 87-101.
[18] Khalaf A. B. and Nehmat K. Ahmed, S_β-open sets and S_β-continuity in topological spaces, Thai J. of Math. Monthly, Vol. 70, No. 1, (2013), 319-335.
[19] Levine N. (1963), Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly, Vol. 70, 36-41.
[20] Levin N. (1970), Generalized closed sets in topology, Rend. Circ. Math. Paleremo Vol. 19, No. 2, 89-96.
[21] Mashhour A. S., Allam A. A., Mahmoud F. S. and Khedr F. H. (1983), On Supra topological spaces, Indian J. Pure and Appl. Math., Vol. 14, No. 4, 502-510.
[22] Mashhour A. S., Hasanein I. A. and El-Deeb S. N. (1982), A note on semi-continuity and precontinuity, Indian J. Pure Appl. Math. Vol. 13, No. 10, 1119-1123.
[23] Njastad O. (1965), On some classes of nearly open sets, Pacific J. Math., Vol. 15, No. 3, 961-970.
[24] Park J. H., Lee B. Y., and Son M. J. (1997), On δ-semiopen sets in topological spaces, J. Indian Acad. Math. 19(1), 59-67.
[25] Reilly I. (2002), Generalized closed sets: a survy of recent work, General and Geometric Topology and its Applications, Vol. 1248, 1-11.
[26] Shareef H. A. (2007), S_p-open sets, S_p-continuity and S_p-compactness in Topological spaces, M. Sc. Thesis, Univ. of Sulaimani.
[27] Sharma, J. (1977), "Topology", Published by Krishna Mandir, Meerut(U. P) and Printed at Manoj printers, Meerut.
[28] Steen L. A. and Seebach J. A. (1978), Counterexamples in Topology, Springer Verlag New York Heidelberg Berlin.
[29] Stone M. H. (1937), Algebraic characterizations of special Boolean rings, Fund. Math., Vol. 29, 223-302.
[30] Veličoko N. V. (1968), H-closed topological spaces, Amer. Math. Soc. Transl., Vol. 78, No. 2, 103-118.
[31] Vishvajit V.S. G. (2001),, Some equivalent of extermally disconnected spaces, Indian J. Pure Appl. Math., Vol. 32, No. 3, 343-344.
[32] Willard S. (1970), General Topology, Addison-Wesley Publishing Company.
Yunis R. H. (2007), Properties of θ-semi-open sets, Zanco J. of Pure and Appl. Sci., Vol. 19, No. 1, 116-122.