



## A new type of nano open set

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### Abstract

The objective of this work is to introduce a new class of nano open set called nano  $S_w$ -open set in nano topological spaces. Basic properties of nano  $S_w$ -open sets are analyzed and compare this nano set with some other nano open sets.

### Key Words:

Nano  $S_w$ -open set,

Nano semi-open set,

Nano topological space

## Introduction

In 2013, Lellis Thivagar proposed the idea of nano topological space, which has been described in terms of approximations and the boundary region of a subset of the universe using an equivalence relation [6]. Lower approximations generate a subset with certain items which will certainly make up part of an interested subset, however upper approximations create a subset with unknown items that may form the basis of an interested subset [2]. The nano open sets are the components of a nano topological space. Nano closed sets, nano interior, and nano closure are further terms he has created. The weaker versions of nano open sets have been developed by him namely (nano  $\alpha$ -open, semi-open, nano pre-open and nano regular open) [6]. Nano  $\beta$ -open sets was introduced by A. Revathy and Gnanambal Ilango in 2015 [4].

## Preliminaries

The aim of this section is to give some definitions, theorems and results that we used in the next sections.

### Definitions 2.1: [2]

Let  $V$  be the universe, which is a non-empty finite collection of things, and  $\Gamma$  be the indiscernibility relation, which is an equivalence relation on  $V$ . The pair  $(V, \Gamma)$  is said to be the approximation space. Let  $X \subseteq V$ :

- (i) Denote  $L_\Gamma(X) = \bigcup_{x \in V} \{ \Gamma(x) : \Gamma(x) \subseteq X \}$ , where  $\Gamma(x)$  denotes the equivalence class determined by  $x$ , the lower approximation of  $X$  with respect to  $\Gamma$ .
- (ii) Denote  $U_\Gamma(X) = \bigcup_{x \in V} \{ \Gamma(x) : \Gamma(x) \cap X \neq \emptyset \}$ , the upper approximation of  $X$  with respect to  $\Gamma$ .

(iii) Denote  $B_\Gamma(X) = U_\Gamma(X) - L_\Gamma(X)$  the boundary region of  $X$  with respect to  $\Gamma$ .

**Property 2.2:** [2] If  $(V, \Gamma)$  is an approximation space and  $X, Y \subseteq V$ , then:

- 1)  $L_\Gamma(X) \subseteq X \subseteq U_\Gamma(X)$ ;
- 2)  $L_\Gamma(\phi) = U_\Gamma(\phi) = \phi$  and  $L_\Gamma(U) = U_\Gamma(U) = U$ .
- 3)  $U_\Gamma(X \cup Y) = U_\Gamma(X) \cup U_\Gamma(Y)$ .
- 4)  $U_\Gamma(X \cap Y) \subseteq U_\Gamma(X) \cap U_\Gamma(Y)$ .
- 5)  $L_\Gamma(X) \cup L_\Gamma(Y) \subseteq L_\Gamma(X \cup Y)$ .
- 6)  $L_\Gamma(X \cap Y) \subseteq L_\Gamma(X) \cap L_\Gamma(Y)$ .
- 7)  $L_\Gamma(X) \subseteq L_\Gamma(Y)$  and  $U_\Gamma(X) \subseteq U_\Gamma(Y)$ , whenever  $X \subseteq Y$ .
- 8)  $U_\Gamma(X^c) = [L_\Gamma(X)]^c$  and  $L_\Gamma(X^c) = [U_\Gamma(X)]^c$ .
- 9)  $U_\Gamma U_\Gamma(X) = L_\Gamma U_\Gamma(X) = U_\Gamma(X)$ .
- 10)  $L_\Gamma L_\Gamma(X) = U_\Gamma L_\Gamma(X) = L_\Gamma(X)$ .

**Definitions 2.3:** [6]

Let  $V$  be the universe,  $\Gamma$  an equivalence relation on  $V$  and  $\tau_\Gamma(X) = \{V, \phi, L_\Gamma(X), U_\Gamma(X), B_\Gamma(X)\}$ , where  $X \subseteq V$ . If  $\tau_\Gamma(X)$  satisfies the following axioms:

- (i)  $V, \phi \in \tau_\Gamma(X)$ .
- (ii)  $\cup_{i \in \Delta} S_i$  is in  $\tau_\Gamma(X)$ , for any subcollection  $\{S_i: i \in \Delta\}$  of  $\tau_\Gamma(X)$ .
- (iii) The finite intersection of members of  $\tau_\Gamma(X)$  is in  $\tau_\Gamma(X)$ .

Then  $(V, \tau_\Gamma(X))$  is termed nano topological space, and the components of  $\tau_\Gamma(X)$  are known as nano open sets, and the dual nano topology of  $\tau_\Gamma(X)$  is known as  $[\tau_\Gamma(X)]^c$  which including nano closed set.

**Definition 2.4:** [6] If  $(V, \tau_\Gamma(X))$  is a nano space with respect to  $X$  and if  $S \subseteq V$ ; then the nano interior of  $S$  ( $Nint(S)$ ) is defined by  $Nint(S) = \cup \{H \in \tau_\Gamma(X): H \subseteq S\}$ ; That is,  $Nint(S)$  is the largest nano open subset of  $S$ . The nano closure of  $F$  ( $Ncl(F)$ ) is defined as  $Ncl(F) = \cap \{K \in [\tau_\Gamma(X)]^c: F \subseteq K\}$ ; That is,  $Ncl(F)$  is the smallest nano closed set containing  $F$ .

**Definition 2.5:** [6] Let  $(V, \tau_\Gamma(X))$  be a nano space and  $A \subseteq V$ . Then  $A$  is:

- (i) Nano semi-open if  $A \subseteq Ncl(Nint(A))$ .
- (ii) Nano pre-open if  $A \subseteq Nint(Ncl(A))$ .
- (iii) Nano  $\alpha$ -open if  $A \subseteq Nint(Ncl(Nint(A)))$ .
- (iv) Nano regular-open if  $A = Nint(Ncl(A))$ .

$NSO(V, X)$ , (resp.  $NPO(V, X)$ ,  $N\alpha O(V, X)$  and  $NRO(V, X)$ ) denote the families of all nano semi-open, (resp. nano pre-open, nano  $\alpha$ -open and nano regular-open) subsets of  $V$ .

**Theorem 2.6:** [8] If  $(V, \tau_\Gamma(X))$  is a nano topological space with respect to  $X$ , where  $X \subseteq V$  and  $A, B$  are subsets of  $V$ , then:

- (i)  $A \subseteq B$  implies that  $Nint(A) \subseteq Nint(B)$ .
- (ii)  $Nint(A) \cup Nint(B) \subseteq Nint(A \cup B)$ .

**Definition 2.7:** [3] A subset  $H$  of a nano topological space  $(V, \tau_\Gamma(X))$  is called nano t-set if  $Nint(Ncl(H)) = Nint(H)$ .

**Proposition 2.8:** [1] Each nano  $\beta$ -open set, which is nano semi-closed in nano topological space  $(V, \tau_\Gamma(X))$  is nano semi-open set.

**Corollary 2.9:** [1] Each nano  $\beta$ -closed set, which is nano  $\alpha$ -open in nano topological space  $(V, \tau_\Gamma(X))$  is nano regular-open set.

**Definition 2.10:** [7] A nano topological space  $(V, \tau_\Gamma(X))$  is nano hyperconnected if the intersection of any two non-empty nano open sets is non-empty.

**Definition 2.11:** [5] A subset  $A$  of a nano topological space  $(V, \tau_\Gamma(X))$  is nano dense if  $Ncl(A) = V$ .

**Theorem 2.12:** [7] In nano hyperconnected topological space  $(V, \tau_\Gamma(X))$ , every non-empty nano open set is nano dense.

**Proposition 2.13:** [1] In nano topological space  $(V, \tau_\Gamma(X))$ , if  $K \in \tau_\Gamma(X)$  and  $L \in NSO(V, X)$ , then  $K \cap L \in NSO(V, X)$ .

**Definition 2.14:** [5] A nano topological space  $(V, \tau_\Gamma(X))$  is nano locally indiscrete if every nano open set is nano closed.

**Proposition 2.15:** [3] A subset  $A$  of a nano topological space  $(V, \tau_\Gamma(X))$  is nano semi-closed if and only if it is a nano t-set.

### Nano $S_w$ -open set

In this section we investigate a new nano open set and gives some of their properties and compare it with some other types of nano open sets.

**Definition 3.1:** Let  $(V, \tau_\Gamma(X))$  be a nano topological space. A subset  $A$  of  $V$  together with the empty set is called nano  $S_w$ -open if  $Nint(A) \neq \phi$ . The family of all nano  $S_w$ -open sets of  $V$  is denoted by  $NS_wO(V, X)$  and  $NS_wC(V, X)$  represent the class of nano  $S_w$ -closed sets in  $V$  which contains the complement of all nano  $S_w$ -open sets.

**Example 3.2:** Consider  $V = \{a, b, c, d\}$ ,  $V \setminus \Gamma = \{\{a, b\}, \{c\}, \{d\}\}$  and  $X = \{b, c\} \subseteq U$ . Then  $\tau_\Gamma(X) = \{V, \phi, \{c\}, \{a, b\}, \{a, b, c\}\}$  is a nano topology on  $V$ . Now if  $A = \{a, b, d\}$ , then  $Nint(A) = \{a, b\} \neq \phi$ , this implies that  $A$  is nano  $S_w$ -open set, while  $B = \{b, d\}$  is not nano  $S_w$ -open set, because  $Nint(B) = \phi$ .

**Lemma 3.3:** In nano topological spaces  $(V, \tau_\Gamma(X))$ , if  $G \in NSO(V, X)$ , then  $G$  is nano  $S_w$ -open.

**Proof:** Let  $A \subseteq V$  be in  $NSO(V, X)$ . Then  $A \subseteq Ncl(Nint(A))$ . Now, if  $A = \phi$ , then it is clear from **Definition 3.1** that  $A$  is nano  $S_w$ -open set, but if  $A \neq \phi$  implies that  $Nint(A) \neq \phi$ . Thus, in both cases  $A$  is nano  $S_w$ -open set.

The converse of **Lemma 3.3** may not be true as in **Example 3.2**, the set  $B = \{a, c\}$  is nano  $S_w$ -open set since, while  $B$  is not nano semi-open set, because  $Ncl(Nint(B)) = \{c, d\}$ , so  $B \not\subseteq Ncl(Nint(B))$ .

**Proposition 3.4:** A subset  $H$  of a nano topological space  $(V, \tau_\Gamma(X))$  is nano  $S_w$ -open if and only if there exists a nano semi-open set  $K$  such that  $K \subseteq H$ .

**Proof:** Let  $H$  be a nano  $S_w$ -open set. Then  $Nint(H) \neq \phi$ , but  $Nint(H) \subseteq H$  and  $Nint(H)$  is a nano open set, which is nano semi-open set, then take  $K = Nint(H)$  this implies that there exists a nano semi-open set  $H$  such that  $K \subseteq H$ .

Conversely: Let the hypothesis be hold. Then for a subset  $H$  of  $V$  there exists a nano semi-open set  $K$  such that  $K \subseteq H$ , then  $Nint(K) \subseteq Nint(H)$  by [**Theorem 2.6 (i)**], but  $Nint(K) \neq \phi$ , implies that  $Nint(H) \neq \phi$  and then  $H$  is nano  $S_w$ -open set in  $V$ .

**Theorem 3.5:** Let  $(V, \tau_r(X))$  be a nano topological space. If  $L_r(X) = U_r(X)$ , then  $V, \phi, L_r(X)$  and any super set of  $L_r(X)$  are nano  $S_w$ -open sets in  $V$ .

**Proof:** Clearly  $\phi, V$  and  $L_r(X)$  are nano  $S_w$ -open sets. Now, let  $A \subseteq V$  such that  $L_r(X) \subseteq A$ , then  $\phi \neq Nint(L_r(X)) = L_r(X) \subseteq Nint(A)$ , this implies that  $Nint(A) \neq \phi$ . Hence  $A$  is nano  $S_w$ -open set.

**Theorem 3.6:** If in a nano topological space  $(V, \tau_r(X))$ ,  $L_r(X) = \phi$ , then  $V, \phi, U_r(X)$  and any subset of  $V$ , which contains  $U_r(X)$  are nano  $S_w$ -open sets.

**Proof:** Clearly  $V, \phi, U_r(X)$  are nano  $S_w$ -open sets.

Now, let  $A \subseteq V$  such that  $U_r(X) \subseteq A$ , then by [Theorem 2.6 (i)],  $\phi \neq Nint(U_r(X)) = U_r(X) \subseteq Nint(A)$  implies that  $A$  is nano  $S_w$ -open set.

**Theorem 3.7:** If  $U_r(X) = L_r(X) = V$  in a nano topological space  $(V, \tau_r(X))$  with respect to  $X$ , then  $V$  and  $\phi$  are the only nano  $S_w$ -open sets.

**Proof:** Let  $U_r(X) = L_r(X) = V$ . Then  $\tau_r(X) = \{\phi, V\}$ , which is indiscrete nano topology on  $V$ . Now, if  $\phi \neq A \subseteq V$ , then  $Nint(A) = \phi$ , this implies that  $A$  is not nano  $S_w$ -open set in  $V$ . Hence  $\phi$  and  $V$  are the only nano  $S_w$ -open sets.

**Theorem 3.8:** If  $U_r(X) = V$  and  $L_r(X) \neq \phi$  in a nano topological space  $(V, \tau_r(X))$ , then  $V, \phi, L_r(X), B_r(X)$  and any subset of  $U$ , which contains  $L_r(X)$  or  $B_r(X)$  are nano  $S_w$ -open sets.

**Proof:** Let  $U_r(X) = V$  and  $L_r(X) \neq \phi$ . Then  $\tau_r(X) = \{V, \phi, L_r(X), B_r(X)\}$ . Now, clearly  $\phi, V, L_r(X)$  and  $B_r(X)$  are nano  $S_w$ -open sets. Let  $\phi \neq A \subseteq V$  such that  $L_r(X) \subseteq A$ , then  $Nint(L_r(X)) = L_r(X) \subseteq Nint(A)$  implies that  $Nint(A) \neq \phi$  and then  $A$  is nano  $S_w$ -open set. If  $B_r(X) \subseteq A$ , then by the same way  $A$  is nano  $S_w$ -open set. Hence  $V, \phi, L_r(X), B_r(X)$  and any subset of  $V$ , which contains  $L_r(X)$  or  $B_r(X)$  are nano  $S_w$ -open sets.

**Proposition 3.9:** If  $A \subseteq V$  is a nano  $S_w$ -open set in nano topological space  $(V, \tau_r(X))$  with respect to  $X \subseteq V$ , then  $Ncl(A)$  is also a nano  $S_w$ -open set.

**Proof:** Let  $A$  be nano  $S_w$ -open set. Then  $Nint(A) \neq \phi$ , but by [Definition 2.4],  $Nint(A) \subseteq A \subseteq Ncl(A)$  and then by [Theorem 2.6 (i)],  $Nint(A) \subseteq Nint(Ncl(A))$ , this implies that  $Nint(Ncl(A)) \neq \phi$ , and then  $Ncl(A)$  is nano  $S_w$ -open set.

**Proposition 3.10:** In nano topological space  $(V, \tau_r(X))$ , the nano closure of every non-empty nano pre-open set is nano  $S_w$ -open.

**Proof:** Let  $A \neq \phi$  be any nano pre-open set in  $U$ . Then  $A \subseteq Nint(Ncl(A))$ , but  $A \neq \phi$  implies that  $Nint(Ncl(A)) \neq \phi$ , and then  $Ncl(A)$  is nano  $S_w$ -open set in  $V$ .

**Proposition 3.11:** The union of two nano  $S_w$ -open sets in nano topological space  $(V, \tau_r(X))$  is nano  $S_w$ -open.

**Proof:** Let  $A, B$  be two nano  $S_w$ -open sets. Then,  $Nint(A) \neq \phi$  and  $Nint(B) \neq \phi$ , and then  $(Nint(A) \cup Nint(B)) \subseteq Nint(A \cup B)$  by [Theorem 2.6 (ii)], but  $(Nint(A) \cup Nint(B)) \neq \phi$ , this implies that  $Nint(A \cup B) \neq \phi$  and then  $A \cup B$  is nano  $S_w$ -open set.

**Remarks 3.12:**

1. The intersection of two nano  $S_w$ -open sets in nano topological space  $(V, \tau_\Gamma(X))$  may not be nano  $S_w$ -open set as example below:  
 Example: Let  $V = \{a, b, c, d\}$  and  $V \setminus \Gamma = \{\{a\}, \{c\}, \{b, d\}\}$  and let  $X = \{a, b\}$ , then  $\tau_\Gamma(X) = \{V, \phi, \{a\}, \{b, d\}, \{a, b, d\}\}$ . Here,  $A = \{a, b\}$  and  $B = \{b, d\}$  are nano  $S_w$ -open sets, while,  $A \cap B = \{b\}$  is not nano  $S_w$ -open set since  $Nint(A \cap B) = \phi$ .
2. If  $A$  is a nano  $S_w$ -open set in  $V$ , then  $A^c = V - A$  may not be nano  $S_w$ -open set as exists in the above Example in **[Remark 1]**, if  $A = \{a, b, d\}$ , then  $A$  is nano  $S_w$ -open set, but  $A^c = \{c\}$  and  $Nint(A^c) = \phi$ , this implies that  $A^c$  is not nano  $S_w$ -open set.

**Proposition 3.13:** In nano topological space  $(V, \tau_\Gamma(X))$ . If  $L_\Gamma(X) \neq U_\Gamma(X)$ , where  $L_\Gamma(X) \neq \phi$  and  $U_\Gamma(X) \neq V$ , then  $V, \phi, L_\Gamma(X), U_\Gamma(X), B_\Gamma(X)$  and any subset of  $V$  which contains  $L_\Gamma(X)$  or  $B_\Gamma(X)$  are nano  $S_w$ -open sets.

**Proof:** Suppose in nano topological space  $(V, \tau_\Gamma(X))$ ,  $L_\Gamma(X) \neq U_\Gamma(X)$ , where  $L_\Gamma(X) \neq \phi$  and  $V_\Gamma(X) \neq V$ . Then,  $\tau_\Gamma(X) = \{V, \phi, L_\Gamma(X), B_\Gamma(X), U_\Gamma(X)\}$ , clearly  $V, \phi, L_\Gamma(X), B_\Gamma(X)$  and  $U_\Gamma(X)$  are nano  $S_w$ -open sets in  $U$ . Now, let  $A \subseteq V$  such that  $L_\Gamma(X) \subseteq A$ . Then  $Nint(A) = L_\Gamma(X) \neq \phi$ , this implies that  $A$  is nano  $S_w$ -open set, and if  $B_\Gamma(X) \subseteq A$ , then it is also nano  $S_w$ -open set. But if  $A \subseteq B_\Gamma(X)$  or  $A \subseteq L_\Gamma(X)$ , then in both case  $Nint(A) = \phi$ , this implies that  $A$  is not nano  $S_w$ -open set.

Thus,  $V, \phi, L_\Gamma(X), U_\Gamma(X), B_\Gamma(X)$  and any subset of  $V$ , which contains  $L_\Gamma(X)$  or  $B_\Gamma(X)$  are nano  $S_w$ -open sets.

**Proposition 3.14:** In nano topological space  $(V, \tau_\Gamma(X))$  every non empty nano t-set and open is nano  $S_w$ -open set.

**Proof:** Let  $\phi \neq A \subseteq V$  be a nano t-set and open set. Then, by **[Definition 2.7]**,  $Nint(Ncl(A)) = Nint(A) = A \neq \phi$ , this implies that  $A$  is nano  $S_w$ -open set.

**Proposition 3.15:** Every non-empty nano regular-open subset of nano topological space  $(V, \tau_\Gamma(X))$  is nano  $S_w$ -open set.

**Proof:** Let  $A \subseteq V$  be a non-empty nano regular-open set. Then  $A = Nint(Ncl(A))$ , implies that  $Nint(A) = A \neq \phi$ , and then  $A$  is nano  $S_w$ -open set.

The converse of **[Proposition 3.15]** may not be true in general as in **Example 3.2**,  $B = \{a, c\}$  is nano  $S_w$ -open set, but not nano regular-open.

**Proposition 3.16:** Each nano  $\beta$ -open, which is nano semi-closed subset of nano topological space  $(V, \tau_\Gamma(X))$  is nano  $S_w$ -open.

**Proof:** Follows from **[Proposition 2.8]** and **[Lemma 3.3]**.

**Corollary 3.17:** Each nano  $\beta$ -closed which is nano  $\alpha$ -open set in a nano topological space  $(V, \tau_\Gamma(X))$  is nano  $S_w$ -open set.

**Proof:** Follows from **[Proposition 3.15]** and **[Corollary 2.9]**.

**Theorem 3.18:** If a nano topological space  $(V, \tau_\Gamma(X))$  is nano hyperconnected space, then the family of nano  $S_w$ -open sets and nano semi-open sets are identical.

**Proof:** In **[Lemma 3.3]**, we have shown that nano semi-open set is nano  $S_w$ -open. Now, if  $A$  is a nano  $S_w$ -open set in  $V$ , then  $Nint(A) \neq \phi$ , but  $(V, \tau_\Gamma(X))$  is nano hyperconnected space, this implies that by **[Theorem 2.12]**,  $Ncl(Nint(A)) = V$ , where  $Nint(A)$  is nano open set, and then  $A \subseteq Ncl(Nint(A))$ , so  $A \in NSO(V, X)$ . Hence  $NSO(V, X) = NS_wO(V, X)$ .

**Corollary 3.19:** In nano hyperconnected topological space  $(V, \tau_r(X))$ , the intersection of nano  $S_w$ -open set and nano open set is nano semi-open set.

**Proof:** Follows from [Theorem 3.18] and [Proposition 2.13].

**Lemma 3.20:** If a nano topological space  $(V, \tau_r(X))$  is nano locally indiscrete, if  $A \in NSO(V, X)$ , then it is nano open.

**Proof:** It is obvious.

**Proposition 3.21:** If a nano topological space  $(V, \tau_r(X))$  is nano hyperconnected and nano locally indiscrete, then every nano  $S_w$ -open set is nano open set.

**Proof:** Follows from [Theorem 3.18] and [Lemma 3.20].

**Proposition 3.22:** Every nano t-set in nano topological space  $(V, \tau_r(X))$  is nano  $S_w$ -closed.

**Proof:** Let  $A$  be a nano t-set in  $V$ . Then by [Proposition 2.15]  $A$  is nano semi-closed set, and then by [Lemma 3.3]  $A$  is nano  $S_w$ -closed.

Finally, we give the following result:

**Proposition 3.23:** A nano topological space  $(V, \tau_r(X))$  is nano hyperconnected if and only if every non-empty nano  $S_w$ -open set is nano dense.

**Proof:** Let  $(V, \tau_r(X))$  be a nano hyperconnected topological space and  $A$  a non-empty nano  $S_w$ -open set. Then  $Nint(A) \neq \emptyset$  and since  $V$  is nano hyperconnected, then by [Theorem 2.12], that  $Ncl(Nint(A)) = V$ . But  $Nint(A) \subseteq A$  implies that  $V = Ncl(Nint(A)) \subseteq Ncl(A)$  and clearly  $Ncl(A) \subseteq V$ , then  $Ncl(A) = V$ . Thus,  $A$  is nano dense.

Conversely: Let the hypothesis be true. To show  $V$  is nano hyperconnected, let  $A$  be a non-empty nano open set, then  $A$  is nano semi-open set and by [Lemma 3.3],  $A$  is nano  $S_w$ -open set, implies that  $Ncl(A) = V$ . Hence every non-empty nano open set is nano dense. Thus,  $V$  is nano hyperconnected.

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