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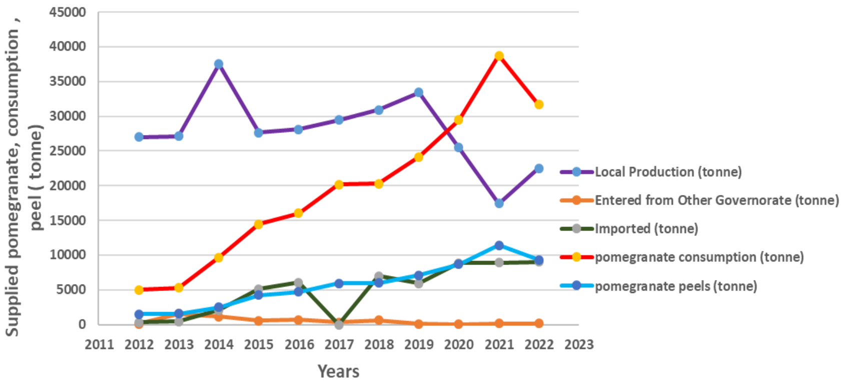

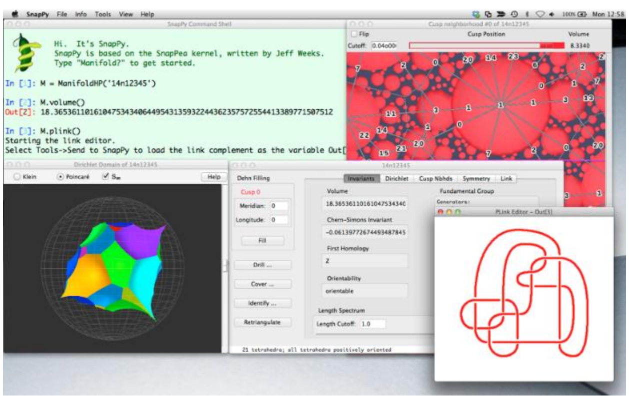

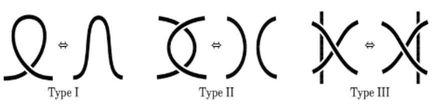

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Converting and using a Novel Approach to Solve a Multi-Objective Linear Programming Problem

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Abstract

We deal with a continuum of alternatives in a finite-dimensional space that are defined by a finite number of linear constraints in Multi-Objective Linear Programming (MOLP). Moreover, there is only one decision maker or decision-making body, and there are a limited number of linear objective functions. To reduce problems with multiple linear objectives to a single linear objective, I employ a novel method. For presentational purposes, numerical samples are given. A comparison is also made between the obtained results and the results of a different strategy that was previously studied.

Introduction

Multi Objective Linear Programming is the study of multiple objective functions that are simultaneously minimized or maximized under a common set of constraints. Selecting target values for each goal is a specific type of goal programming issue. Linear programming is a quantitative technique for allocating limited resources among conflicting tasks. Due to applications in many functional areas such as production, finance, marketing, distribution, advertising and so on, it can justifiably be considered the most popular operational research technique. The popular Chandra Sen methodology [1] has been applied to resolve the Problem of Multi Objective Linear Programming. numerous approaches of solving these issues have been investigated. For example, joão paula [2] investigated the use of a fractional objective function in linear programming in 2005. In 2006, Sulaiman with Sadiq [3] solved these issues utilizing the mean and median values. Sulaiman and Abdul-Qader, [4] developed an optimal transformation technique in 2008, to find a solution for the Multi Objective Linear Programming issue. The Multi Objective Linear fractional programming problem was solved in 2010 by Sulaiman and Salih [5] through the use of mean and median. A novel approach to the transforming and resolving Multi Objective Linear fractional programming problems was proposed by Sulaiman, Gulnar and Basiya [6] in 2014. In 2016, Sulaiman with Rebaz [7] applied the harmonic mean to resolve Problems involving multi objective linear programming. In 2017, Akhtar and

Modi [8] Solving multi objective linear fractional programming issue and compared results with other techniques. A unique statistical averaging technique wa presented by Samsun and Abdul Alim [9] in 2017 to address the Multi Objective Linear Programming problem. In 2022, novel techniques were employed by Snoor and Ronak [10] to solve multi objective linear fractional programming problem. In 2022, Suganya and Afrin [11] used statistical averaging to solve MOLPP. In order to solve the Multi Objective Linear Programming Problem in 2023, Samsun, Marin, and Abdul Alim [12] employed the statistical averaging method in conjunction with the fuzzy programming method.

The goal of this study was to convert a multi-objective linear programming problem into a single-objective problem by proposing a novel technique then, solving it utilize Lingo Model Software. Also, I compared the outcomes using approaches from previous research.

Materials and Methods

MOLPP expressed Mathematically

The following is the mathematical expression for the MOLPP:

$$\left.\begin{aligned} Max.Z_1 &= \beta_1^t x + \alpha_1 \\ Max.Z_2 &= \beta_2^t x + \alpha_2 \\ &. \\ &. \\ &. \\ Max.Z_k &= \beta_k^t x + \alpha_k \\ Min.Z_{k+1} &= \beta_{k+1}^t x + \alpha_{k+1} \\ &. \\ &. \\ &. \\ Min.Z_n &= \beta_n^t x + \alpha_n \end{aligned}\right\} \quad \ldots (1)$$

$$\text{Subject to: } Ax \begin{Bmatrix} \leq \\ \geq \\ = \end{Bmatrix} b \quad \ldots (2)$$

$$x \geq 0 \quad \ldots (3)$$

In this case x described a vector function n of the decision variables, β is vector of size n with constants, b is an $m-$dimensional vector of constants, while A is a $(m \times n)$ coefficient matrix. Of a total of k maximization objective functions, $(n-k)$ minimization objective functions exist, with n being the number of goal functions that simultaneously need to have maximized and minimized. All vectors in this paper are assumed to be column vectored unless transposed (t), for a particular $i = 1, 2, .., n,$ it goes that α_i is a constant in scaler form.

Chandra Sen's method of solving (MOLPP) Methodology:

Here, I follow Sen's approach [1] to formulate the constraint objective function for the MOLPP. Let us assume that the objective functions of the MOLPP in equation (1) yield a single value for each of them. The following constraints (2) and (3) are imposed on them as they are each being optimized independently:

$$\left.\begin{array}{l} Max.Z_1 = \mathcal{L}_1 \\ Max.Z_2 = \mathcal{L}_2 \\ \quad . \\ \quad . \\ \quad . \\ Max.Z_k = \mathcal{L}_k \\ Min.Z_{k+1} = \mathcal{L}_{k+1} \\ \quad . \\ \quad . \\ \quad . \\ Min.Z_n = \mathcal{L}_n \end{array}\right\} \qquad \dots (4)$$

The objective functions' values are represented by $\mathcal{L}_i$, $i = 1, 2, \dots, n$, when there are conflicts between objectives, not every optimal solution will have the same level of the decision variable. To choose the best compromise solution, however, common sets of decision variables between objective functions are required. Utilizing Chandra Sen's method to resolve MOLPP, which takes the following form:

$$Max.Z = \sum_{i=1}^{k} \frac{Z_i}{|\mathcal{L}_i|} - \sum_{i=k+1}^{n} \frac{Z_i}{|\mathcal{L}_i|} \qquad \dots (5)$$

And use simplex methods to solve (5).

* Pearson 2 skewness coefficient: is computed the multiplying the dissimilarity between the mean and median by three. The standard deviation is used to divide the result.

Note) To avoid a negative outcome, incorporated an absolute value into the algorithm, as the values of 2 Pearson's coefficient of skewness can be either positive or negative.

Applying Novel Approach to Address MOLPP:

Using the absolute of Pearson 2 skewness coefficient method, I proposed the combined objective function (1) to resolve the Multi-Objective Linear Programming problem. This function is based on the notion that the combined objective function (4) should be formulated similarly to the formula (5) for MOLPP.

$$Max.Z = \frac{S_1 - S_2}{|S_{k2}|} \qquad \dots (6)$$

Where the Pearson 2 skewness coefficient is denoted by S_{k2} is defined:

$$|S_{k2}| = \left| 3\,\frac{mean|\mathcal{L}_i| - median|\mathcal{L}_i|}{SD} \right|,$$

$S_1 = \sum_{i=1}^{k} Z_i$, for all $i = 1, 2, \dots, k$, $S_2 = \sum_{i=k+1}^{n} Z_i$, for all $i = k+1, \dots, n$.

for every $i = 1, 2, \dots, n.$, the objective function value is denoted by $\mathcal{L}_i$. sample standard deviation is represented by the letter SD.

A special case: Will appear when the mean and median are equal. In this case the algorithm does not work. This indicates a zero skewness (a perfectly symmetrical distribution)

The Pearson 2 skewness coefficient formula has the following algorithm:

The Following summarizes a method for determining the optimal solution for the MOLPP given in equation (4):

Step1: All the distinct goal functions that need to be increasing or decreasing can be given any random value.

Step2: Use the simplex approach to solve the first objective function within the given constraints.

Step3: Before proceeding to to step 4, confirm that step 2's solution is feasible

Step4: For the same value of $i = 1, 2, \dots, n$, put $\mathcal{L}_i$ to the objective function's optimal value·

Step5: Determine $|S_{k2}| = \left|3\frac{mean|\mathcal{L}_i| - median|\mathcal{L}_i|}{SD}\right|$, for each $i = 1, 2, \dots, n$

Step6: Build the combined objective function, whose formula (6) occurs.

Step7: Use the same restrictions (2) and (3) to maximize the combined goal function.

Results and Discussion

Numerical Illustrations

Example 1) Consider the MOLP Problem

$$Max. Z_1 = 2x_1 + x_2$$

$$Max. Z_2 = 3x_1$$

$$Max. Z_3 = 4x_1 - x_2$$

$$Max. Z_4 = x_1$$

$$Min. Z_5 = -3x_1 - x_2$$

$$Min. Z_6 = -10x_1 - x_2$$

Subject to: $x_1 + x_2 \geq 3$; $2x_1 + x_2 \leq 4$; $x_1, x_2 \geq 0$

Solution 1) Utilizing the values of the objective functions listed in Table 1, apply the simplex method to each objective function under the same constraints

Table 1. The result of each objective function solved.

No.	(x_1, x_2)	$Max.Z$	$\lvert\mathcal{L}_i\rvert$	μean $\lvert\mathcal{L}_i\rvert$	Median $\lvert\mathcal{L}_i\rvert$	S.D.
1	(1, 2)	4	4	4.5	3.5	3.937
2	(1, 2)	3	3			
3	(1, 2)	2	2			
4	(1, 2)	1	1			
5	(1, 2)	-5	5			
6	(1, 2)	-12	12			

$$S_1 = \sum_{i=1}^{4} Max.Z_i = 10x_1; \qquad S_2 = \sum_{i=5}^{6} Min.Z_i = -13x_1 - 2x_2$$

Solve example (1) **By Pearson 2 skewness coefficient:**

$$|S_{k2}| = \left|3\frac{mean|\mathcal{L}_i| - median|\mathcal{L}_i|}{SD}\right| = 0.769,$$

$$Max.Z = \frac{S_1 - S_2}{S_{k2}} = \frac{23x_1 + 2x_2}{0.769}$$

Subject to: $x_1 + x_2 \geq 3;$ $\quad 2x_1 + x_2 \leq 4;$ $\quad x_1, x_2 \geq 0$

Utilize Lingo and the simplex method to solve it, we get, $\boldsymbol{Max.Z = 35.1105\ at\ (1, 2)}$

When Example (1) is solved using a different approach, the outcomes are:

By Cahndra Sen., $Max.Z = 5.93x_1 + 0.034x_2$, Once the identical constraints are applied, the solution is obtained. $\boldsymbol{Max.Z = 5.998}$**, at (1,2).**

Using Mean, $Max.Z = \frac{23x_1+2x_2}{4.5}$, solve it having the same constraints the outcome is, $\boldsymbol{Max.Z = 6}$**, at (1,2).**

By Median,$Max.Z = \frac{23x_1+2x_2}{3.5}$, solve it having the identical constraints get, $\boldsymbol{Max.Z = 7.71}$**, at (1,2).**

Utilizing arithmetic mean, $Am_1 = 2.5$, $\qquad Am_2 = 8.5$,

$Max.Z = \frac{S_1}{Am_1} - \frac{S_2}{Am_2} = 5.5x_1 + 0.2x_2$ obtained $\boldsymbol{Max.Z = 5.9}$**, at (1,2)**, when solve it with the same constraints.

Using new arithmetic mean, $m_1 = \min(\mathcal{L}_i|)_{Max} = 1$, $\qquad m_2 = \min(\mathcal{L}_i|)_{Min} = 5$, $Nam = \frac{m_1+m_2}{2} = 3$

$Max.Z = \frac{S_1 - S_2}{Nam} = \frac{23x_1 + 2x_2}{3}$, with the same constraints solve it the outcome is, $\boldsymbol{Max.Z = 9}$, **at (1,2).**

By Quadratic mean, $Qm_1 = 2.74$, $Qm_2 = 9.19$,

$Max.Z = \frac{S_1}{Qm_1} - \frac{S_2}{Qm_2} = 5.06x_1 + 0.22x_2$, solve it, by applying the same constraints, $\boldsymbol{Max.Z = 5.5}$, **at (1,2).**

By New Quadratic mean, $m_1 = \min(\mathcal{L}_i|)_{Max} = 1$, $m_2 = \min(\mathcal{L}_i|)_{Min} = 5$, $NQm = 3.6$

$Max.Z = \frac{S_1 - S_2}{NQm} = \frac{23x_1 + 2x_2}{3.6}$, resolve problem while maintaining the same restrictions, get $\boldsymbol{Max.Z = 7.5}$, **at (1,2).**

Example 2) Consider the MOLP Problem

$Max.Z_1 = x_1$

$Max.Z_2 = x_1 + 2x_2 + 2$

$Max.Z_3 = x_2 + 3$

$Min.Z_4 = -3x_2$

$Min.Z_5 = -x_1 - 3x_2$

Subject to: $2x_1 + 3x_2 \leq 6$; $x_1 \leq 4$; $x_1 + 2x_2 \leq 2$; $x_1, x_2 \geq 0$

Solution 2) Illuminate each objective function utilizing the simplex procedure with the same confinements, utilizing the values from Table 2's objective function values.

Table 2. The results of each objective function solved

No.	**(x_1,x_2)**	**$Max.Z$**	**$\|\mathcal{L}_i\|$**	**μean $\|\mathcal{L}_i\|$**	**Median $\|\mathcal{L}_i\|$**	**S.D.**
1	(2,0)	2	2	3.2	3	0.837
2	(2,0)	4	4			
3	(0,1)	4	4			
4	(0,1)	-3	3			
5	(0,1)	-3	3			

$$S_1 = \sum_{i=1}^{3} Max.Z_i = 2x_1 + 3x_2 + 5; \qquad S_2 = \sum_{i=4}^{5} Min.Z_i = -x_1 - 6x_2$$

Utilizing the formula **(6)**, Solve example (2)**:**

$$|S_{k2}| = \left|3\frac{mean|\mathcal{L}_i| - median|\mathcal{L}_i|}{SD}\right| = 0.717,$$

$$Max.Z = \frac{S_1 - S_2}{|S_{k2}|} = \frac{3x_1+9x_2+5}{0.717}$$

Subject to: $2x_1 + 3x_2 \leq 6$; $x_1 \leq 4$; $x_1 + 2x_2 \leq 2$; $x_1, x_2 \geq 0$

Maximum can be obtained by applying the simplex method and (Lingo Model Software) $\boldsymbol{at}\ (\mathbf{0}, \mathbf{1})$ $\boldsymbol{Max.Z = 19.526}$

Example 3) Consider the MOLP Problem [11]

$Max.Z_1 = x_1 + 2x_2$

$Max.Z_2 = x_1$

$Min.Z_3 = -2x_1 - 3x_2$

$Min.Z_4 = -x_2$

Subject to: $6x_1 + 8x_2 \leq 48$; $x_1 + x_2 \geq 3$; $x_1 \leq 4$; $x_2 \leq 3$; $x_1, x_2 \geq 0$

Solution 3) Table 3 presents the results of applying the Simplex method to solve the goal functions.

Table 3. The result of solution of each goal function's solution

No.	(x_1,x_2)	$Max.Z$	$\lvert\mathcal{L}_i\rvert$	μ**ean** $\lvert\mathcal{L}_i\rvert$	**Median** $\lvert\mathcal{L}_i\rvert$	**S.D.**
1	(4,3)	10	10	8.5	7	6.45497
2	(4,0), (4, 3)	4	4			
3	(4,3)	-17	17			
4	(0,3), (4, 3)	-3	3			

$S_1 = \sum_{i=1}^{2} Max.Z_i = 2x_1 + 2x_2$; $S_2 = \sum_{i=3}^{4} Min.Z_i = -2x_1 - 4x_2$

Using equation (6) to solve example (3)

$|S_{k2}| = \left|3\frac{mean|\mathcal{L}_i| - median|\mathcal{L}_i|}{SD}\right| = 0.697,$

$Max. Z = \frac{S_1 - S_2}{|S_{k2}|} = 5.73888x_1 + 8.608x_2$

Subject to: $6x_1 + 8x_2 \leq 48;$ $x_1 + x_2 \geq 3;$ $x_1 \leq 4;$ $x_2 \leq 3;$ $x_1, x_2 \geq 0$

After applying the simplex method and Lingo Model Software to solve it, we obtain, $\boldsymbol{Max. Z = 48.78\ at\ (4, 3)}$

Example 4) Consider the MOLP Problem

$Max. Z_1 = 5x_1 + x_2 + 3x_3$

$Max. Z_2 = 3x_1 + 2x_2$

$Min. Z_3 = -x_1 - x_2 - x_3$

$Min. Z_4 = -x_1 - x_3$

Subject to: $x_1 + x_2 + x_3 \leq 2;$ $2x_1 - 3x_2 \leq 6;$ $x_1, x_2, x_3 \geq 0$

Solution 4) Table 4 presents the results of applying the Simplex method to solve the goal functions.

Table 4. The result of solution of each goal function's solution

No.	(x_1, x_2)	$Max. Z$	$\lvert\mathcal{L}_i\rvert$	μean $\lvert\mathcal{L}_i\rvert$	Median $\lvert\mathcal{L}_i\rvert$	S.D.
1	(2, 0,0)	10	10	5.5	5	3.416
2	(2, 0,0)	6	6			
3	(2, 0,0)	-2	2			
4	(2, 0,0)	-4	4			

$S_1 = \sum_{i=1}^{2} Max. Z_i = 8x_1 + 3x_2 + 3x_3;$ $S_2 = \sum_{i=3}^{4} Min. Z_i = -3x_1 - x_2 - 2x_3$

Using equation (6) to solve example (4)

$|S_{k2}| = \left|3\frac{mean|\mathcal{L}_i| - median|\mathcal{L}_i|}{SD}\right| = 0.439,$

$Max. Z = \frac{S_1 - S_2}{|S_{k2}|} = \frac{8x_1 + 3x_2 + 3x_3 - (-3x_1 - x_2 - 2x_3)}{0.439}$

Subject to: $x_1 + x_2 + x_3 \leq 2;$ $2x_1 - 3x_2 \leq 6;$ $x_1, x_2, x_3 \geq 0$

After applying the simplex method and Lingo Model Software to solve it, we obtain, $\boldsymbol{Max.Z} = \mathbf{50.1139}\ \boldsymbol{at}\ (\mathbf{2}, \mathbf{0}, \mathbf{0})$

And solved examples (2, 3 & 4) by another method that showed in Table 4.

Comparing the suggested approach with other approaches

In Table 4, present the numerical results of the comparison between the other methods and the suggested technique.

Table 4. Evaluating various numerical methods' results

Techniques	Example 1	Example 2	Example 3	Example 4
Chandra Sen. Technique	5.998	4	3.9998	5
Mean Technique	6	4.375	4	4
Median Technique	7.71	4.667	4.8569	4.4
Arithmetic Average Technique	5.9	4.4	3.9999	4
New Arithmetic Average Technique	9	5.6	9.7143	5.5
Quadratic mean	5.5	4.3	3.49	3.838
New Quadratic mean	7.51	5.6	9.62	4.919
Pearson 2 skewness coefficient	35.1105	19.526	48.78	50.1139

Table 4 makes it clear that the **Pearson 2 skewness coefficient** technique yielded better results than the Mean and Median, Chandra Sen, Arithmetic Average, New Arithmetic Average, Quadratic mean and New Quadratic Mean techniques in Examples **1, 2, 3 and 4.**

Conclusion

This article discussed various approaches that have been investigated to find the optimal MOLPP solutions. Specifically, I introduced a brand-new method for handing the Multi-Objective Linear Programming problem (MOLPP), which involves converting the MOLPP to SLPP using the Pearson 2 skewness coefficient method. In contrast, give some examples of these approaches, which are predicated on the significance of the objective functions. In solving above examples clear that the technique that are used for converting the MOLLPP into SOLPP is superior than the other approaches that were previously researched. More efficiently than earlier methods, the Pearson 2 skewness coefficient method solves the problem.

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